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Identification of the heat transfer coefficient during cooling process by means of Trefftz method

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ABSTRACT

In the article, the Trefftz method, belonging to the analytical-numerical methods, was used to identify the heat transfer coefficient for Inconel during spray cooling at the part of boundary. A mathematical model of the process has been formulated using cylindrical coordinate. The heat transfer coefficient is identified based of the Robin condition at the cooled boundary. Trefftz functions for heat conduction equation in cylindrical dimensionless coordinates has been used to find an approximate solution of the problem. As the internal temperature responses the real data from measurements has been used. The result of calculation is similar to the obtained by the other method.

1. Introduction

In the metallurgical industry, cooling by liquid spray is used to improve the quality of products in forging, heat treatment or welding processes [1–10]. The parameter that plays a key role in controlling the cooling process is the heat transfer coefficient [1,4,5,9,11–15]. This factor affects the size of the heat flux density exchanged with the environment and is related to changes in the temperature field of the cooled material.

In heat transfer problems, the temperature field is unambiguously determined by the differential equation of heat transfer, boundary conditions and the initial condition [16]. The issues in which these conditions are known and, in addition, thermo-physical parameters are prescribed as well as the area in which the temperature field is examined, are called direct problems. However, in most cases during cooling processes, the measurement of temperature or heat flux on a part of the boundary is not possible, and in addition the heat transfer coefficient is there often unknown. Then, in order to supplement the data needed to determine the temperature field inside and on the whole boundary of the cooled body, temperature measurements are often used at selected internal points (so called internal temperature responses, abbr. ITRs). The issue of determining the temperature field and the heat transfer coefficient on the part of the boundary based on these measurements and the initial conditions and boundary conditions known on the remaining part of the boundary is called the boundary inverse problem. It belongs to the class of ill-posed problems in the Hadamard sense, [17–21]. They are characterized by the fact that slight inaccuracies in ITRs measurements may cause very large inaccuracies of solutions. In other words, a large change in the heat transfer coefficient on the boundary may be accompanied by a slight change in temperature at the measurement location.

These types of problems were considered using many methods. Use of analytical methods is usually limited to one-dimensional problems and simple geometrical forms, it is usually assumed that the thermal properties of the material are constant, [17,22–27], and many others. The great progress in solving inverse problems resulted from the use of various types of regularization methods, from the Tichonov regularization, [21,28], to physical regularization, [29,30]. This usually led to the combined analytical-numerical methods, with emphasis on numerical. Much greater opportunities arise from the use of strictly numerical methods [1,11,31–34], although then each one input data must be examined separately and it is difficult to draw general conclusions. To solve the inverse problems finite element method (FEM) is one of the most commonly used. These method similarly to the other numerical ones requires uncertainty analysis including eg special test function, which answers the question what is the accuracy of the model used in calculation [35]. Frequently the finite difference method [4,36–38] or finite volume method [39] has been used. Interesting results were also brought by Beck's method of sensitivity coefficients [17,40]. Attempts to use neural networks were also made [41]. Good results are also achieved by using the Picard method [42,43] and homotopy perturbation method, [44,45], also applicable to non-linear inverse problems [46].

One of the most useful methods of solving an inverse problem is the method of approximating a solution of the problem with a linear

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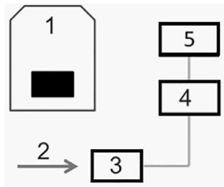

Fig. 1. Test stand. 1 – electric furnace, 2 – spray nozzle (2 MPa), 3 – cylindrical sample, 4 – temperature measurement system, 5 – personal computer.

combination of Trefftz fuctions (T-functions). The Trefftz method is widely described in many papers and monographs and articles, e.g., Refs. [18–20,47–58]. The T-functions for a linear partial differential equation usually can be obtained by the method of variables separation. Expanding the solution(s), called generating function(s), into power series with respect to the parameter(s) resulting from the method of variable separation one arrives to T-functions. In 2000, two new methods of deriving the T-functions have been published [18].The one based on expanding function in Taylor series is particularly simple and effective; the second is based on the so-called inverse operators [18,59]. The T-functions used in this article have been derived in [60].

In the article, the Trefftz method, belonging to the analytical-numerical methods, was used to identify the heat transfer coefficient. A mathematical model of the process has been formulated using cylindrical coordinate. The heat transfer coefficient is identified based of the Robin condition at the cooled boundary. Trefftz functions for heat conduction equation in cylindrical dimensionless coordinates has been used to find an approximate solution of the problem. The real data from measurements, [15], has been used to complete the conditions. In order for them to have values that are physically acceptable, they have been smoothed out using a combination of Trefftz functions. The results of calculation are similar to the ones obtained by the other method, [61].

2. Brief description of the experiment

In the papers [15,61], the experiment in which the cylindrical test sample has been sprayed with water was described. The layout of the test stand is shown in Fig. 1. During the experiment, the heat flow in the cylindrical sample was close to one-dimensional. The results of the tests presented in [11] have shown that the dimensions of such a built-up cylindrical sample are of secondary importance in the identification of the heat transfer coefficient. The diameter and height of the cylinder were equal to 20 mm. Cylindrical surface of the sample and one of its bases were surrounded by a screen made of the same material as the sample material. The sample was made of Inconel. The space between the sample and the screen was 5 mm and was filled with air. This way, the convective heat transfer between the sample and the screen could be neglected. The identification of the heat transfer coefficient was based on the results of temperature measurements at three points along the axis of the cylinder. The distribution of temperature measurement points was determined both by technical considerations and the need to obtain a “readable” temperature difference between neighboring points during cooling. It was decided that the first measuring point would be located at a distance of 2 mm from the cooled face, and the next two measuring points would be located at a distance of 4 and 6 mm from the cooled face, respectively, Fig. 2. K-type thermocouples, 80 μm in diameter, placed in a 0.5 mm diameter cover were used for temperature measurement. The water spray temperature was 20 °C. The results of measurements were obtained from the authors of the paper [61].

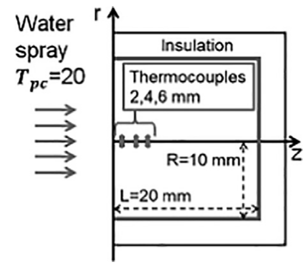

Fig. 2. Thermocouples location in the sample.

3. Mathematical model of the problem and the input data

To describe the temperature field in the sample the heat conduction equation in cylindrical variables can be written:

$$\lambda\left[\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial T(\bar r,\bar z,\bar t)}{\partial \bar r}\right)+\frac{\partial}{\partial \bar z}\left(\frac{\partial T(\bar r,\bar z,\bar t)}{\partial \bar z}\right)\right]-\rho c_p\frac{\partial T(\bar r,\bar z,\bar t)}{\partial \bar t}=0, \tag{1}$$

where $0<\bar r<R=0.01$ [m], $0<\bar z<L=0.02$ [m] (see Fig. 2), $\bar t>0$ [s] stands for time, λ, [W/(mK)], is the heat conduction coefficient, ρ, [kg/m^3] is density, c_p, [J/(kgK)], denotes specific heat and T, [°C], describes temperature of the sample. The conditions at the boundaries $\bar r=R$, $\bar z=0$ and $\bar z=L$ and can be expressed as follows:

$$\frac{\partial T(R,\bar z,\bar t)}{\partial \bar r}=0, \tag{2}$$

$$\frac{\partial T(\bar r,L,\bar t)}{\partial \bar z}=0, \tag{3}$$

$$\frac{\partial T(\bar r,0,\bar t)}{\partial \bar z}=\alpha_{z=0}(\bar t)\left(T_{pc}-T(\bar r,0,\bar t)\right) \tag{4}$$

where T_{pc}, [°C], stands for the temperature of the spray water. The initial condition describes the temperature of the sample after removal from the furnace:

$$T(\bar r,\bar z,0)=800\,^{\circ}\mathrm{C} \tag{5}$$

The boundary condition (4) can be transformed into a form showing the dependence of the heat transfer coefficient on temperature of the spray water and temperature of the cooled surface:

$$\alpha_{z=0}(\bar t)=\frac{\partial T(\bar r,0,\bar t)}{\partial \bar z}\frac{\lambda}{T_{pc}-T(\bar r,0,\bar t)} \tag{6}$$

Temperatures indicated by thermocouples (i.e., temperature internal responses, abbr. ITRs) have been measured for $\bar z=0.002, 0.004, 0.006$ m and $\bar t=0(0.01)32.9$ s (i.e., in 3290 moments of time). The results of these measurements are subject to errors. These can be errors in the installation of thermocouples resulting from, for example, from dependence of thermophysical properties on temperature, initial temperature distribution in the sample, measurement errors and accidental errors, small fluctuations in power supply. Therefore, the curves describing exactly (according to measurements) the temperature values at the measuring points can be incompatible with any solution of Eq. (1). For this reason, the results of the measurements will be further smoothed using the Trefftz functions (T-function). These functions strictly meet Eq. (1), so that the measured data subjected to such smoothing will be compatible with a solution of Eq. (1).

Average properties of sample made of Inconel are as follows:

– specific heat $c_p=449.8\,\mathrm{J/(kgK)}$,
– thermal conductivity $\lambda=14.467\,\mathrm{W/(mK)}$,
– density$\rho=8470\,\mathrm{kg/m^3}$,
– characteristic dimension $L=0.02\,\mathrm{m}$.