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Topology optimization of steel frame structures with constraints on overall and individual member instabilities

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ABSTRACT

A computationally efficient structural topology optimization framework is proposed for design of steel frame structures with user-defined factors of safety against overall structure (global) and individual member instabilities. The objective function is minimization of either compliance or the maximum of von Mises stresses within the frame structure. Within optimization, overall structure buckling modes are found via an eigenvalue analysis, a subset of "pseudo modes" are identified using a newly proposed methodology and are discarded to obtain a set of real eigenvalues. Moreover, individual member buckling loads are estimated with Euler buckling analysis and are aggregated into a single constraint. The minimum of each instability constraint is then estimated with separate differentiable negative p-norm functions. Sensitivities of these newly developed constraints are explicitly derived for application of gradient-based optimizers. The topology of four frame structures featuring moment-resisting connections and member cross-sectional properties mapped from the American Institute of Steel Construction design manual are optimized with the proposed algorithm to verify its effectiveness in optimizing structural performance while maintaining factors of safety against overall and individual member instabilities. The interaction effects of preventing instabilities at different safety levels and the choice of objective function on the final designs and their performances are investigated.

1. Introduction

Structural topology optimization (STO) is rapidly making inroads as a design tool in structural engineering for identification of optimized material distributions for trusses (pinned connections) and frame structures (moment-resisting connections). This technique is more general than other structural optimization approaches because it allows for member sizes and topological features of the design (e.g., member connectivity) to change simultaneously within the iterative design process [1]. STO is often performed with the "ground structure" approach. The ground structure is a dense mesh of candidate structural members, which serves as the initial guess for the optimizer. At every optimization iteration, a finite element analysis is performed to quantify the structural performance, i.e., determining the objective function and design constraints. Moreover, a sizing optimization is performed to optimize the objective function (gradient-based methods in this work) while the constraints are satisfied. At the end of these optimization iterations, members with low cross-sectional areas (below a prescribed threshold) are removed from the design domain to obtain a cleaned-up version of the topology. Because this cleaning up results in a (small) loss of material, another design iteration could be performed using this cleaned-up structure as the initial guess, and this process is continued until convergence is reached. Therefore, changes in the topology as well as member sizes are achieved.

The ground structure approach is successfully implemented in minimum compliance topology STO under a prescribed volume of material, where novel designs with very high stiffness are developed (see e.g., [2,3] and the references therein). However, achieving realistic designs requires many other practical structural performance aspects to be directly incorporated within STO. Incorporation of these objectives (or constraints) has generated significant research interest, but at the same time has posed numerical and theoretical challenges. One area of intense research is development of computationally efficient stress-based topology optimization algorithms, which proved to be significantly more challenging than compliance-based topology optimization (e.g., the design singularity problem; see Refs. [4–11] and references therein). Another area of active research is preventing structural instabilities (or buckling) within the STO process, which is the focus of what follows.

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The equilibrium of a structure (or any member) under compression loads might be an unstable equilibrium. This means that at higher load magnitudes any imposed perturbation on the structure (e.g., an admissible deflection) may lead the structure to follow a different equilibrium path that is accompanied with large and sudden deformations (buckling). Therefore, STO should recommend member sizes and connectivity so that these buckling limit-states do not occur before other strength or serviceability limits (e.g., yield stress in members). Early STO algorithms were focused primarily on truss structures, and controlled only for individual member buckling through limiting their axial stresses to be under the member Euler buckling stress capacity [12–14]. The stress singularity problems and formation of disjoint feasible domains were primarily tackled with the ϵ-relaxation method of Refs. [7,15]. However, this approach of limiting the axial stress in every member introduced a large number of constraints (as many as number of members), and quickly became computationally expensive and even infeasible for enlarged ground structures. Therefore, only ground structures with a small number of members were optimized in these works.

Including measures only against individual member buckling in STO does not guarantee the overall structure safety. For instance, a chain of collinear members can still be formed, which is highly sensitive to overall structural buckling. Refs. [17] and [18] were the first authors to note this, and suggested adding system-level stability constraints or geometric imperfections (as a measure for overall structural buckling) in topology optimization. Including geometric imperfections for probabilistic compliance-based design of trusses is discussed in Refs. [4,5,19,20], and was further developed to include a direct methodology for overall structure buckling control in Ref. [21]. Overall structure buckling capacity can also be determined with solving an eigenvalue problem under linear elastic conditions. Currently, applications are restricted to truss topology optimization, [22–27]. For each design (either the final design or any intermediate one), the number of buckling modes (eigenvectors), and their corresponding load factors (eigenvalues) can be as many as the number of degrees-of-freedom. Controlling the minimum buckling load factor at every optimization iteration leads to a topology that achieves a user-defined level of safety against the overall structural buckling limit state. Note that conducting a geometrically nonlinear analysis, where the equilibrium equation is written in residual form and is solved iteratively using a variant of Newton-Raphson or Riks (arc-length) method, might converge to an unstable equilibrium branches where the tangent stiffness is negative. Refs. [28] suggest that an eigenvalue type constraint is still needed to ensure a stable equilibrium regime is achieved for the final design using geometric nonlinear analysis.

Implementation of an eigenvalue analysis within a gradient-based STO poses the following two challenges: 1) identification of "pseudo modes" of buckling and 2) determination of the minimum eigenvalue with a differentiable function [29–31]. It is noted that unlike sizing optimization, where member connectivity is fixed, topology optimization allocates material freely to all candidate structure members. Therefore, at each optimization iteration, there will be regions of the structure with relatively low volume of material, and regions with a larger share of the material. The buckling modes that mainly activate the degrees-of-freedom in the low area regions of the design are termed pseudo modes of buckling. Because the main contribution in the design performance comes from members with relatively higher cross-sectional areas, these pseudo-modes should be identified and eliminated before determining the minimum load factor. Methodologies to eliminate the pseudo modes is proposed in Refs. [32,33] for vibration problems. These approaches are based on modifying element stiffness matrix and/or mass matrix in low-density regions. However, buckling eigenvalue analysis requires construction of the geometric stiffness matrix of each element, which is a function of its current axial stresses. Therefore, pseudo mode identification within STO is more complicated and requires specific methodologies to be developed.

The present paper proposes an efficient methodology for compliance- and stress-based topology optimization of frame structures (with moment-resisting connections) under overall and individual member buckling constraints. This article is structured as follows. Section 2 reviews the minimum compliance and stress-based design for frame structures using sections from the AISC design manual [34]. Section 3 proposes a new methodology for pseudo-mode identification (specific to frame structures), which is integrated within an eigenvalue analysis for overall structural stability. Moreover, the minimum buckling load factor is accurately estimated using a p-norm function. The next focus of Section 3 is on determining individual member buckling through controlling their Euler elastic buckling loads. Here, a new constraint is proposed to aggregate the resulting constraints into a single one, thereby increasing computational efficiency. An efficient method for sensitivity analysis of the newly proposed design constraints is presented that allows using gradient-based optimizers in Section 4, which also presents the solution algorithm. Four numerical examples are solved in Section 5, which show the effectiveness of the proposed methodology to optimize stress or compliance-based design objectives while controlling instabilities. Moreover, the designs are compared and it is shown that both overall and individual member buckling constraints are required to ensure design safety. Section 6 offers concluding remarks.

2. Compliance-based and stress-based topology optimization of steel frames

This section reviews the minimum compliance (compliance-based) and stress-based topology optimization under linear elastic conditions.

2.1. Compliance-based design

We begin with the well-known minimum compliance topology optimization of frames, which is defined as follows:

$$\min_{\mathbf{a}} C = \mathbf{f}^T\mathbf{d}(\mathbf{a}) \tag{1}$$

$$s.t.\ \ \mathbf{K}(\mathbf{a})\mathbf{d}(\mathbf{a}) = \mathbf{f}$$

$$\mathbf{a}^T\mathbf{l} \le v$$

$$a_{\min} < a_e \le a_{\max}$$

where boldface lower and upper case letters symbolize vectors and matrices respectively. In the above formulation, the objective function is the compliance C. The first constraint is used to enforce static equilibrium under the force vector $\mathbf{f}$. The global stiffness matrix is denoted with $\mathbf{K}$ and the displacement vector is shown with $\mathbf{d}$. The design variables are member cross-sectional areas stored in the vector $\mathbf{a}$, with an individual member of a_e. The other cross-sectional properties (e.g., moment of inertia) are mapped from the AISC design manual with the approach described in Refs. [4,5]. The member lengths are collected in the vector $\mathbf{l}$, and the total amount of available volume of material is denoted by v. Therefore, the third constraint limits the available volume. Furthermore, $a_{\min}$ is a small quantity that limits minimum cross-sectional area to avoid singularity of the stiffness matrix, and $a_{\max}$ is the maximum allowable cross-sectional area. These two establish the bounds on the design variables as expressed in the third constraint. Computational efficiency is achieved via using gradient-based optimizers with the sensitivity provided in Refs. [4,5].

2.2. Stress-based design

The designs using the objective function in Eq. (1) are sought so that a measure of structural stiffness (inverse of compliance) is maximized. In structural engineering applications, stresses (usually von