Finite Elements in Analysis and Design 140 (2017) 38–49

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Finite element analysis of plasma dust-acoustic waves

P. Areias [a,c,*], J.N. Sikta [a,b], M.P. dos Santos [a]

[a] Department of Physics, University of Évora, Colégio Luís António Verney, Rua Romão Ramalho, 59, 7002-554, Évora, Portugal
[b] Department of Physics, Jahangirnagar University, 1342 Savar, Dhaka, Bangladesh
[c] CERIS/Instituto Superior Técnico, University of Lisbon, Portugal

ARTICLE INFO

Keywords:
Dusty plasma
Acoustic waves
Vortex
Finite elements
Nonlinear fluid

ABSTRACT

For dust acoustic solitary waves, we propose a finite element formulation of the fluid dusty plasma equations. To solve this continuum problem, a Petrov-Galerkin weak form with upwinding is applied. We consider an unmagnetized dusty plasma with negatively charged dust and Boltzmann distributions for electrons and ions. Nonlinearity of ion and electron number density as functions of the electrostatic potential is included. A fully-implicit time-integration is used (backward-Euler method) which requires the derivative of the weak form. A three-field formulation is introduced, with dust number-density, electrostatic potential and dust velocity being the unknown fields. We test the formulation with two numerical (2D and 3D) examples where convergence with mesh size is assessed. These establish the new formulation as a predictive tool in dusty plasmas.

1. Introduction

In the 1920's, Irving Langmuir [1] proposed that electrons, ions and neutrals in an ionized gas can be, as a whole, considered a corpuscular material, he titled as plasma. It is now accepted that more than 99% of the known universe, in which the dust is a omnipresent ingredient, is in the plasma state [2,3]. Plasmas containing dust particles are important in the study of the space environment, such as asteroid zones, planetary rings (viz. Saturn rings [4]), comet tails, as well as Earth's magnetosphere [5]. In addition, dusty plasmas are observed in laboratory within Q-machines, DC discharges, and RF discharges. Dusty plasmas typically contain dust grains of micrometre or sub-micrometre size which are negatively charged by field emission, ultra-violet ray irradiation, and plasma currents [6,7]. Collective effects in micro-plasmas have been studied by Verheest [8] using many-fluid models.

The presence of grains of dust alters existing plasma wave spectra and introduces dust-acoustic waves, dust ion-acoustic waves, dust lattice waves etc. [9]. In dust acoustic waves, inertia is provided by dust particle mass and the restoring force is provided by the pressures of electrons and ions. Dust acoustic waves predicted by Rao, Shukla and Yu have been experimentally observed by Barkan et al. [10]. As mentioned by Merlino and Goree [11], in a dust acoustic wave neighbor, dust fluid elements are coupled by the electric field associated with the wave rather than by collisions, as they would be in a neutral gas.

We are not aware of antecedent finite element solutions of dusty plasma. However, two-fluid finite element solutions of plasmas exist, one significant solution has been developed by C.R. Sovinec's group, cf. [12,13] using classical (continuous) Galerkin methods. Inherent instabilities caused by the convective term are dealt at the time-integration level, cf. [13]. In the work of Jardin, Breslau and Ferraro [14], the Clough-Tocher $\mathscr{C}^1$ triangular element was used to solve smooth magneto-hydrodynamic problems. Other solutions exist for magneto-hydrodynamics, cf. [15,16], which, although related to this work, are a parallel development. If the analysis involves shock waves, discontinuous Galerkin methods (cf. [17]) have been used with success with plasmas, see Levy, Shu and Yan [18].

The main goal of this first work is to create a computational framework, with a set of benchmarks, for the analysis of dusty plasma dynamics. More ingredients (magnetic field, Newton viscosity, non-Newtonian effects) will be introduced as found relevant, on top of thoroughly tested software.

Herein, we assume that dust particles have constant mass and are point charges. In addition, we consider a three component plasma consisting of electrons and ions having Boltzmann density distributions with temperatures T_e and T_i, respectively, as well as negatively charged, heavy dust particles. To simplify the treatment, thermal motion of dust is not included. This case has been identified as "cold dust" by Rao, Shukla and Yu [7]. In dust-free electron–ion plasmas, ions charge generally remains constant. However, in a dusty plasma, the charge of a particle does not remain constant, cf. [11]. With the goal of obtaining a stable solution of a shock-free problem, we opt for an implicitly integrated Petrov-Galerkin formulation, which results in a very simple but effec-

* Corresponding author. Department of Physics, University of Évora, Colégio Luís António Verney, Rua Romão Ramalho, 59, 7002-554, Évora, Portugal.
E-mail address: pmaa@uevora.pt (P. Areias).

https://doi.org/10.1016/j.finel.2017.10.010
Received 16 March 2017; Received in revised form 24 October 2017; Accepted 27 October 2017
Available online 22 November 2017
0168-874X/© 2017 Elsevier B.V. All rights reserved.

Table 1
Relevant quantities and constants (cf. 5–15).

e	Electron charge $[1.6022 \times 10^{-19}$ C]
κ_B	Boltzmann constant $[1.38065 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}]
ε_0	Vacuum permittivity $[8.85419 \times 10^{-12}$ Fm^{-1}]
Z_d	Dust charge number $Z_d = -Q_d/e$
n_{d0}	Dust number density at equilibrium $[\mathrm{m}^{-3}]$
n_{i0}	Ion number density at equilibrium $[\mathrm{m}^{-3}]$. Approximation: $n_{i0} \cong Z_d n_{d0}$
n_{e0}	Electron number density at equilibrium $[\mathrm{m}^{-3}]$. Approximation: $n_{e0} \cong n_{i0}$
T_e	Electron temperature [K]
T_i	Ion temperature [K]
Z_i	Ion charge number $Z_i = Q_i/e$
m_d	Mass of a dust particle [Kg]
$\boldsymbol{u}_0(\boldsymbol{x})$	Initial dust velocity
n_{d0}	Initial number density of dust
$\overline{\boldsymbol{u}}(t)$	Imposed velocity at the boundary Γ_u
$\overline{n}(t)$	Imposed density at the boundary Γ_n
$\overline{\varphi}(t)$	Imposed electrostatic potential at the boundary Γ_φ
$\overline{t}(t)$	Imposed electrostatic gradient at the boundary $\Gamma_{\varphi'}$

tive numerical scheme. We organize this work as follows: in Section 2 the governing equations are presented (continuity, momentum and Poisson), with the respective initial and boundary conditions. In Section 3, the weak form using a Petrov-Galerkin combination of test/trial functions is presented. This is followed, in Section 4, by the discretization using Streaming Upwind Petrov-Galerkin (SUPG) [19] shape functions. In Section 5, a set of 2D and 3D representative numerical examples is shown, confirming the robustness in terms of mesh and step-size effects in the numerical results. Finally, in Section 6, conclusions are drawn.

2. Governing equations

2.1. Characteristic quantities

We consider the following independent unknown fields:

- n_d: number density of dust
- $\boldsymbol{u}_d$: dust velocity
- φ: electrostatic wave potential

Equations for a dusty plasma typically make use of normalized quantities, which in turn depend on characteristic values. We introduce the Debye length for a dusty plasma, using the ion temperature, as (see the approximation $\lambda_d \cong \lambda_i$ in Ref. [3]):

$$\lambda_i = \frac{1}{Z_i e}\sqrt{\frac{\varepsilon_0 \kappa_B T_i}{n_i}} \tag{1}$$

where ε_0 is the electric permittivity of free space, κ_B is the Boltzmann constant, T_i is the ion temperature (with Kelvin units), Z_i is the ion charge number, n_i is the number density of ions and e is the electron charge. In this work, ions are positively charged and dust is negatively charged. In addition, λ_i serves as a characteristic length-scale. In terms of time, we introduce thermal speed of dust, v_d, which is obtained from the dust temperature T_d and the dust particle mass m_d as:

$$v_d = \sqrt{\frac{\kappa_B T_d}{m_d}} \tag{2}$$

The dust acoustic speed (c_{da}) for cold dust is given by (cf. [9]):

$$c_{da} = \sqrt{\frac{Z_d \kappa_B T_i}{m_d}} \tag{3}$$

where Z_d is the dust charge number. Making use of equilibrium (*initial*) quasi-neutrality, we have:

$$n_{i0} Z_i = n_{e0} + n_{d0} Z_d \tag{4}$$

Table 2
Properties in use for the numerical examples (ionosphere [25]).

Z_d	1000
n_{d0}	1×10^8 m^{-3}
n_{i0}	3×10^{11} m^{-3}
n_{e0}	2×10^{11} m^{-3}
T_e	1160.45 K
T_i	1160.45 K
m_d	1.05893×10^{-12} kg
λ_e	0.00429109 m
λ_i	0.00429109 m
T^d	6.93282 s
$\varphi^\star$	0.0999597 NmC^{-1}

where $Z_i = 1$ considered in the remainder of this work. In (4), n_{i0}, n_{e0} and n_{d0} are the ion, electron and dust number densities for $t = 0$, which is identified as equilibrium time.

2.2. Fluid theory of the dust acoustic wave: differential equations and boundary conditions

Dust is considered cold, $T_d \ll T_i$ in the following equations. The domain under consideration is denoted by Ω and the time interval under consideration is $[0, T]$. The governing equations for these unknown fields are, given that $\boldsymbol{x} \in \Omega$ and $t \in [0, T]$,

Continuity equation on $\Omega \times [0, \mathrm{T}]$:

$$\dot{n}_d + \nabla \cdot \left(n_d \boldsymbol{u}_d\right) = 0 \tag{5}$$

Momentum equation on $\Omega \times [0, \mathrm{T}]$:

$$n_d m_d \dot{\boldsymbol{u}}_d + n_d m_d \left(\nabla \boldsymbol{u}_d\right) \cdot \boldsymbol{u}_d + m_d c_{da}^2 \nabla n_d - n_d e Z_d \nabla \varphi = 0 \tag{6}$$

Poisson-like equation on $\Omega \times [0, \mathrm{T}]$:

$$\nabla^2 \varphi + \frac{e}{\varepsilon_0}\left(n_i - n_e - n_d Z_d\right) = 0 \tag{7}$$

see Ref. [7]. These equations are complemented by the initial and boundary conditions for the unknown functions $\boldsymbol{u}_d(\boldsymbol{x}, t)$, $n_d(\boldsymbol{x}, t)$ and $\varphi(\boldsymbol{x}, t)$:

$$\boldsymbol{u}_d(\boldsymbol{x}, 0) = \boldsymbol{u}_0(\boldsymbol{x}) \tag{8}$$

$$n_d(\boldsymbol{x}, 0) = n_{d0}(\boldsymbol{x}) \tag{9}$$

Download English Version:

https://daneshyari.com/en/article/6925480

Download Persian Version:

https://daneshyari.com/article/6925480

Daneshyari.com