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Topology optimization of binary structures using Integer Linear Programming

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ABSTRACT

This work proposes an improved method for gradient-based topology optimization in a discrete setting of design variables. The method combines the features of BESO developed by Huang and Xie [1] and the discrete topology optimization method of Svanberg and Werme [2] to improve the effectiveness of binary variable optimization. Herein the objective and constraint functions are sequentially linearized using Taylor's first order approximation, similarly as carried out in [2]. Integer Linear Programming (ILP) is used to compute globally optimal solutions for these linear optimization problems, allowing the method to accommodate any type of constraints explicitly, without the need for any Lagrange multipliers or thresholds for sensitivities (like the modern BESO [1]), or heuristics (like the early ESO/BESO methods [3]). In the linearized problems, the constraint targets are relaxed so as to allow only small changes in topology during an update and to ensure the existence of feasible solutions for the ILP. This process of relaxing the constraints and updating the design variables by using ILP is repeated until convergence. The proposed method does not require any gradual refinement of mesh, unlike in [2] and the sensitivities every iteration are smoothened by using the mesh-independent BESO filter. Few examples of compliance minimization are shown to demonstrate that mathematical programming yields similar results as that of BESO for volume-constrained problems. Some examples of volume minimization subject to a compliance constraint are presented to demonstrate the effectiveness of the method in dealing with a non-volume constraint. Volume minimization with a compliance constraint in the case of design-dependent fluid pressure loading is also presented using the proposed method. An example is presented to show the effectiveness of the method in dealing with displacement constraints. The results signify that the method can be used for topology optimization problems involving non-volume constraints without the use of heuristics, Lagrange multipliers and hierarchical mesh refinement.

1. Introduction

The methods for structural topology optimization have been under intense research during the last couple of decades. The ultimate goal of topology optimization is to obtain binary solutions representing optimal structural layouts. The topology optimization problem can be modeled using binary variables, 0 and 1 representing the void and solid regions of the structure, respectively. This type of binary variable optimization presents an extremely challenging large-scale integer programming problem and alternatives to this formulation were proposed [4].

Towards obtaining 0/1 structures, the first widely accepted ideas considered topology optimization of continuum structures with the relaxation of binary constraint $\{0,1\}$ by using continuous density design variables [5]. This transforms the binary variable optimization problem into a density distribution problem, used in the homogenization-based method and the SIMP (Solid Isotropic Material with Penalization) method [6,7]. The SIMP model rapidly became a popular material interpolation based method with the work by Ref. [8] and his role in dissemination of the method [9]. The method was successfully applied to solve a range of important problems, such as design for nonlinear responses [10–12], stress-based design [13], design for fluid flow [14,15], dynamics design [16] and others. In such methods, gray transition material regions are intrinsically allowed between solid and void in the continuous variables definition. This leads to optimal solutions with non-explicitly defined structural boundaries and, despite their popularity, this is challenging the development of these methods in problems where explicit boundary description is important, e.g., in design-dependent multiphysics problems.

Some researchers proposed few techniques to reduce or eliminate intermediate density (gray scale) elements in the final solutions, such as projection methods. These techniques consider Heaviside functions [17,18] or morphology-based operators [19,20] to project filtered densities into 0/1 solution space while aiming length scale control.

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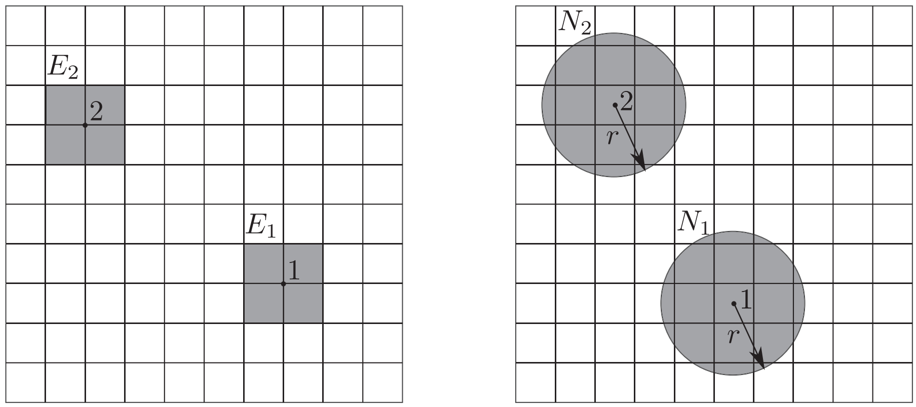


(a) Nodes 1 and 2 (b) Elements 1 and 2

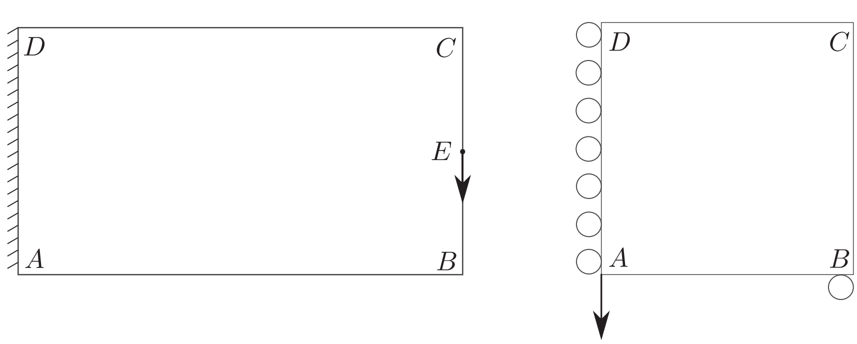
Fig. 1. Filtering - Areas of averaging for nodal and filtered elemental sensitivities.

There are some gradient-based methods created for obtaining 0/1 optimal solutions. One class consists of boundary-description based methods, e.g. the level-set method (LSM), in which an implicit function is used to describe the structure with clearly defined boundaries. In these methods, the design variables are boundary points and shape sensitivities are derived to predict design changes [21–23]. The optimization is based on the structural shape movements and, thus, the final solution is strongly affected by the initial design configuration. A novel method with explicit boundary representation was developed by Christiansen et al. [24] with the combination of shape and topology optimization. Other class of gradient-based methods that obtain 0/1 designs employs a discrete approach, which is the focus of this paper.

The most established discrete topology optimization method is the Bi-directional Evolutionary Structural Optimization (BESO) [25]. The idea is to switch variables between void and solid using a design update scheme where sensitivities serve as indicators of the design variable performance. Although the idea was initially proposed by Xie and Steven [26], the method as it is currently used was developed by Huang and Xie [1], presenting convergent and mesh-independent solutions. Comprehensive reviews on the BESO methods are given in Refs. [27,28]. The method has been applied to a wide range of problems like nonlinear structures [29], natural frequency maximization [30,31], material optimization and multiscale problems [32–37], multiphysics problems [38,39], etc. The design variables are updated based on the thresholds of sensitivity numbers corresponding to the objective function, and the thresholds are set based on the evolutionary ratios. In this work, the use of mathematical programming enables to update the design variables without using any thresholds. Another discrete topology optimization method was proposed by Svanberg and Werme [40] where the authors effectively proposed a sequential integer linear programming approach, where one starts with a coarse mesh to solve an optimization problem and uses the final solution of this problem as the initial solution for optimization on a refined mesh and so on. They also fix a region of the structure every iteration to speed up the optimization [2]. The method proposed in this work uses a BESO filter to smoothen the sensitivities, which makes it robust and removes the need for hierarchical mesh refinement.

One more discrete structural optimization method uses Genetic Algorithms. These are derivative-free techniques based on natural selection. The design variables are genetically encoded and a pool of solutions are heuristically updated over generations [41,42]. There are too many ways of designing these algorithms and convergence is not always guaranteed. The heuristics used for cross over and selection are problem-dependent and greatly affect convergence.

While density-based methods are very well developed but do not present explicit boundaries during optimization, the BESO method showed effective potential as a binary approach such as in problems where boundary identification is important, e.g., in fluid-structure interaction problems [43]. The modern BESO method [1] updates the design variables relying on a fixed change in volume fraction every iteration, and is not based on mathematical optimization. Thus, it is not guaranteed that each iteration of BESO is an optimal step. The early ESO/BESO methods solved volume minimization problems with stress [26], displacement and frequency constraints [44]. These methods are based on heuristics [3] and are non-convergent [1]. The modern BESO method uses Lagrange multipliers to deal with non-volume constraints. The selection and updating of these multipliers is not trivial. This paper aims to create an improved discrete topology optimization method by



(a) Cantilever with vertical load (b) Michell Beam

Fig. 2. Design domains and their loading configurations.