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Spectral representation of stochastic field data using sparse polynomial chaos expansions

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ABSTRACT

Uncertainty quantification is an emerging research area aiming at quantifying the variation in engineering system outputs due to uncertain inputs. One approach to study problems in uncertainty quantification is using polynomial chaos expansions. Though, a well-known limitation of polynomial chaos approaches is that their computational cost becomes prohibitive when the dimension of the stochastic space is large. In this paper, we propose a procedure to solve high dimensional stochastic problems with a limited computational budget. The methodology is based on an existing non-intrusive model reduction scheme for polynomial chaos representation, introduced by Raisee et al. [1], that is further extended by introducing sparse polynomial chaos expansions. Specifically, an optimal stochastic basis is calculated from a coarse scale analysis, using proper orthogonal decomposition and sparse polynomial chaos and is then utilized in the fine scale analysis. This way, the computational expense on both the coarse and fine discretization levels is drastically reduced. Two application examples are considered to validate the proposed method and demonstrate its potential in solving high dimensional uncertainty quantification problems. One analytical stochastic problems is first studied, where up to 20 uncertainties were introduced in order to challenge the proposed method. A more realistic CFD type application is then discussed. It consists of a two dimensional NACA 0012 symmetric profile operating at subsonic flight conditions. It is shown that the proposed reduced order method based on sparse polynomial chaos expansions is able to predict statistical quantities with little loss of information, at a cheaper cost than other state-of-the-art techniques.

1. Introduction

Uncertainty quantification (UQ) is the process of describing the stochastic behavior of outputs of interest due to uncertain inputs. Many problems in engineering and science are subject to uncertainties on input variables and design variables. An example would be to predict the aerodynamic forces acting on an airplane operating at specific flight conditions. Small differences in airspeed, atmospheric conditions, as well as manufacturing tolerances on airplane components will lead to different predictions, which may have disastrous consequences if they are not accounted for from the premises of the industrial design process. This involves identifying and quantifying relevant uncertainties, modeling and incorporating them into a non-deterministic methodology.

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Nevertheless, the inclusion of uncertainties in real-life engineering problems is, to date, lagging behind. The main reasons are the complexity and cost of computational models describing real-life problems, e.g. Computational Fluid Dynamics (CFD) or Finite Element (FE) solvers, in conjunction with the often high number of uncertain parameters, all leading to high computational cost. It is therefore crucial to develop a strategy to propagate uncertainties in presence of a large number of parameters.

The Polynomial Chaos (PC) method has received much attention recently in a wide range of applications [2–6]. The method is based on an expansion of the stochastic solution in terms of orthogonal polynomials in the space of stochastic variables. PC was formulated first by Wiener [7] to model stochastic processes governed by Gaussian-shaped random variables. It was later extended by Xiu and Karniadakis [8,9] to various distributions using orthogonal polynomials from the so-called Askey scheme [10]. This extension is often referred to as *generalized* PC. The main advantage of PC methods is their higher statistical convergence rates, provided that the response quantity is smooth [11].

PC approaches can be classified as either intrusive or non-intrusive. In intrusive PC, the PC expansion is directly integrated in the governing equations describing the problem. Applications of intrusive PC to CFD problems exist [12–16] but we rather focus on non-intrusive methods as they require no modification to the existing solver and are therefore more attractive from an engineering point of view. In non-intrusive methods, the deterministic solver is indeed utilized as a separate entity (black-box) that is called at specific points selected in the stochastic space. This results in a linear system of equations which can then be solved to compute the PC coefficient values. However, the number of samples grows exponentially with the number of uncertainties. This is a well-known limitation of standard PC methods, often referred to as the *curse-of-dimensionality*.

To mitigate this curse, a variety of stochastic computational methods have been proposed in the literature [17–19]. A recurring idea to speed-up the process has been to exploit different level of discretizations. Major computational savings can be achieved by distributing the workload between different meshes, as shown e.g. for non-intrusive PC based on collocation and regression [20,21]. This concept has been successfully applied to aerodynamic problems where important CPU savings were observed compared to an approach based on a single high-fidelity expansion.

Methods based on reduced basis decomposition of the stochastic field and PC expansions have also been proposed in [1,22–28]. In particular, an interesting idea, proposed by Doostan et al. [23], is to construct an optimal stochastic basis with inexpensive calculation on a coarse grid and then use this basis for the fine scale analysis. The method was first successfully applied by Doostan et al. [23] to 2D solid mechanics problems using intrusive PC, where the number of unknowns was reduced from 165 to 5 using a 3rd order PC. It was later extended in [1,22] to non-intrusive PC using a regression approach and was mostly validated on CFD-based applications. Advantages of the method are twofold: important CPU savings can be achieved with little loss in accuracy and it also gives access to complete stochastic field information, which is a great analytical tool. However, the approach still requires the construction of a full PC expansion on the coarse grid, which in some case could turn out to be very complicated or even impossible.

In a recent past, adaptive methods [17,29] have also emerged as a remedy for solving high-dimensional stochastic problems. These methods rely upon the assumption that the model output can be reasonably well approximated using very few polynomial functions. In most applications, only a few parameters are responsible for response variability. This observation led to the concept of *sparsity* in the full PC expansion, i.e. only a limited number of PC coefficients are different from zero. With this in mind, regression-based sparse PC methods [17,29] have been developed and attempt to detect sequentially the most important PC terms using only few samples. Promising results were obtained for high dimensional solid mechanics problems as well as CFD-based problems, where important CPU savings were demonstrated compared to classical PC solution schemes. To the best of the authors' knowledge, these adaptive methods are only applied on single high-fidelity response models.

In this paper, a computationally efficient methodology is devised for solving high dimensional stochastic problems. A Proper Orthogonal Decomposition (POD)-based model reduction scheme, combined with non-intrusive sparse PC, is developed. Relative to the current state-of-the-art, we propose an extension of [1,22] by deriving the optimal stochastic basis on a coarse grid using sparse PC expansions. In comparison with the aforementioned work, the main advantage of the proposed method lies in reducing significantly the workload on the coarse grid, specially for problems exhibiting a large number of uncertainties. The method is non-intrusive and enables to achieve great savings in terms of CPU cost compared to classical PC solution schemes. Also, the proposed methodology is applicable to field data, which are often encountered in many engineering applications. It is first validated using an analytical benchmark example. A CFD-based problem is also shown to demonstrate the power of the proposed methodology when applied to a more practical application.

This paper is organized as follows. Section 2 gives a brief description of the proposed methodology. It emphasizes the major challenges and introduces the concept of sparse PC expansion and its coupling with proper orthogonal decomposition. Numerical examples are given in Section 3 to showcase the performance of the proposed methodology.

2. Methodology

This section describes the theoretical background of the proposed methodology. We present an extension of the stochastic model reduction scheme introduced in [1] by constructing the covariance structure using adaptive sparse PC expansions.