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A review of level-set methods and some recent applications

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ABSTRACT

We review some of the recent advances in level-set methods and their applications. In particular, we discuss how to impose boundary conditions at irregular domains and free boundaries, as well as the extension of level-set methods to adaptive Cartesian grids and parallel architectures. Illustrative applications are taken from the physical and life sciences. Fast sweeping methods are briefly discussed.

1. Introduction

Representing and tracking the evolution of interfaces is a fundamental component of computer simulations. An efficient way to do so is to use the level-set method introduced by Osher and Sethian [1]. It consists in representing the interface as the level-set of a higher dimensional function. The main advantage of this implicit representation of a moving front is its ability to naturally handle changes in topology, as illustrated in Fig. 1. This is in contrast to explicit methods [2–6] for which changes in topology require extra work for detecting and subsequently treating numerically the merging or pinching of fronts. We note, however, that explicit methods have the advantage of accuracy (e.g. front-tracking preserve volumes better than level-set methods for the same grid resolution) and we refer the interested reader to the work of [7] for a front-tracking method that handle changes in topology. Volume of fluid methods also adopt an implicit formulation using the volume fraction of one phase in each computational cells (see e.g. [8–19] and the references therein). These methods have the advantage of conserving the total volume by construction. They are however more complicated than level-set methods in three spatial dimensions and it is difficult to compute accurately smooth geometric properties such as curvatures from the volume fraction alone, although we refer the reader to the interesting work of Popinet on this issue [20]. Also, we note that phase-field models have been extensively used to tackle free boundary problems, particularly in the case

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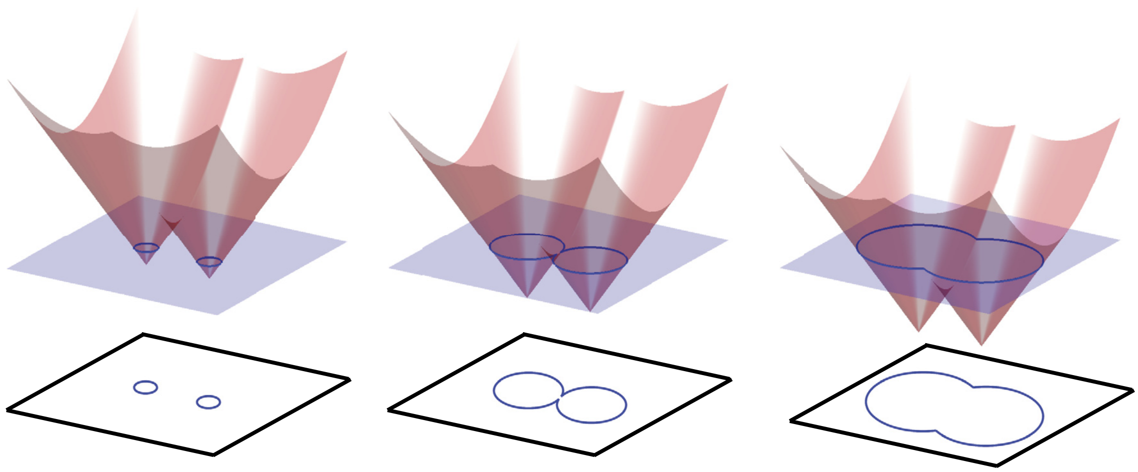

Fig. 1. Level-set representation of a free boundary (blue solid line) in two spatial dimensions, moving in its normal direction, and subsequent changes in topology that are handled automatically. The level-set function is depicted in red. (Color online.)

of solidification processes [21–29]. However, these models do not represent the interface in a sharp fashion, which in turn leads to a degradation of the accuracy where it matters most and impose sometimes stringent time step restrictions. In what follows, we review the level-set method, including the treatment of boundary conditions in that framework and extensions to adaptive Cartesian grids and parallel architectures. The Fast Sweeping Method, which is often associated with the level-set method for its ability to compute the signed distance function and solutions to other Hamilton–Jacobi equations, is also briefly discussed. We then present some recent applications of the level-set and the fast sweeping methods.

2. Level-set representation and equations

The level-set method of Osher and Sethian [1] represents an interface, Γ, (i.e. a curve in two spatial dimensions or a surface in three spatial dimensions) as the zero-contour of a higher dimensional function, ϕ, called the level-set function, which is defined as the signed distance function to Γ:

$$\phi(\boldsymbol{x}) = \begin{cases} -d & \text{for } \boldsymbol{x} \in \Omega^-, \\ +d & \text{for } \boldsymbol{x} \in \Omega^+, \\ 0 & \text{for } \boldsymbol{x} \in \Gamma, \end{cases}$$

where d is the Euclidian distance to Γ. The level-set function can also be used to compute the normal to the interface $\boldsymbol{n}$ and the interface's mean curvature κ:

$$\boldsymbol{n} = \nabla\phi/|\nabla\phi| \qquad \text{and} \qquad \kappa = \nabla \cdot \boldsymbol{n}.$$

Under a velocity field $\boldsymbol{v} = (u, v, w)$, the interface deforms according to the level-set equation:

$$\frac{\partial \phi}{\partial t} + v_n|\nabla\phi| = 0, \qquad \text{where } v_n = \boldsymbol{v} \cdot \boldsymbol{n}. \tag{1}$$

Although the level function can be chosen to be any Lipschitz continuous function, in practice the signed distance function is chosen for its properties of improved mass conservation and accuracy, especially in the computations of geometrical quantities. Since in general the level-set function does not retain its signed-distance-function property as it evolves in time through equation (1), Sussman et al. [30] introduced the reinitialization equation:

$$\phi_\tau + \mathtt{sgn}(\phi^0)\,(|\nabla\phi| - 1) = 0, \tag{2}$$

to transform a level set function $\phi^0 : \mathbb{R}^n \to \mathbb{R}$ into the signed distance function ϕ. Here, $\mathtt{sgn}$ is a smoothed-out signum function and τ represents a fictitious time that controls the width of the band around the zero-level set where ϕ will be sign-distanced. Finally, if the level-set function is a signed distance function, the projection onto Γ of any given point $\boldsymbol{x}$ where $\nabla\phi(\boldsymbol{x})$ is well-defined, is easily computed as:

$$\boldsymbol{x}_\Gamma = \boldsymbol{x} - \phi(\boldsymbol{x})\nabla\phi(\boldsymbol{x}).$$

3. Approximation of equations

On uniform grids, the level-set advection equation (1) and the reinitialization equation (2) are discretized with a HJ-WENO scheme in space [31–33] and a TVD-RK3 in time [34]. We gives the details of those schemes next.