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A time domain sampling method for inverse acoustic scattering problems

Yukun Guo [a], Dietmar Hömberg [b], Guanghui Hu [b], Jingzhi Li [c,*], Hongyu Liu [d]

[a] Department of Mathematics, Harbin Institute of Technology, Harbin, PR China
[b] Weierstrass Institute, Berlin, Germany
[c] Department of Mathematics, Southern University of Science and Technology, Shenzhen, PR China
[d] Department of Mathematics, Hong Kong Baptist University, Kowloon, Hong Kong SAR, PR China

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ABSTRACT

This work concerns the inverse scattering problems of imaging unknown/inaccessible scatterers by transient acoustic near-field measurements. Based on the analysis of the migration method, we propose efficient and effective sampling schemes for imaging small and extended scatterers from knowledge of time-dependent scattered data due to incident impulsive point sources. Though the inverse scattering problems are known to be nonlinear and ill-posed, the proposed imaging algorithms are totally "direct" involving only integral calculations on the measurement surface. Theoretical justifications are presented and numerical experiments are conducted to demonstrate the effectiveness and robustness of our methods. In particular, the proposed static imaging functionals enhance the performance of the total focusing method (TFM) and the dynamic imaging functionals show analogous behavior to the time reversal inversion but without solving time-dependent wave equations.

1. Introduction

The inverse scattering theory of acoustic waves and the corresponding reconstruction methods have been extensively studied in recent years. Considerable developments have been achieved to the inverse scattering problem of determining the location and shape of a scattering object from the knowledge of acoustic scattered waves corresponding to time-harmonic incident waves. Those methods are usually referred to as the frequency-domain methods in the literature; see, e.g., [3,10,20] and the references therein.

Among various frequency-domain methods for inverse scattering problems, we are particularly interested in the so-called sampling-type methods. The core of a sampling-type method is a certain imaging/indicator functional, which is obtained by using the measurement data and can be used to indicate whether a point (or a line, or a surface) in the space belongs to the interior or exterior of the scatterer. In general, the imaging functionals are easily computed without iterations required, though the inverse scattering problems are known to be nonlinear and ill-conditioned. Moreover, the sampling-type methods usually require very little a priori information of the unknown/inaccessible target scatterer. Due to their practical

* Corresponding author.
E-mail addresses: ykguo@hit.edu.cn (Y. Guo), Dietmar.Hoemberg@wias-berlin.de (D. Hömberg), hu@wias-berlin.de (G. Hu), li.jz@sustc.edu.cn (J. Li), hongyuliu@hkbu.edu.hk (H. Liu).

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importance, the sampling-type methods have drawn a great deal of interest in the literature, and examples include the linear sampling method [9], the factorization method [20], the point source and probe method [27], the enclosure method [16], the MUSCI-type method [2,3] and the recent one-shot and orthogonality sampling methods [23,24,28], among others. Generally, the frequency-domain sampling methods could work with the data corresponding to only a single frequency.

However, in diverse practical applications such as sonar detection, geophysical exploration, medical imaging and nondestructive testing, dynamic measurement data are usually easy to obtain. Therefore, imaging unknown/inaccessible scatterers through the time-dependent acoustic waves is a topic of significant interest to these areas. There exist a number of reconstruction algorithms in the time domain such as the time reversal techniques [11,13,14], the reversed time migration [4], the boundary control method [5,26]. In a series of recent publications, such as [6,7,29,19] and references cited there, two globally convergent numerical methods for inverse scattering problem in the time domain were proposed. In this approach the spatially distributed speed $c(x)$, $x \in \mathbb{R}^3$ of the acoustic signal is unknown. Furthermore, the global convergence theory was firmly confirmed on experimental data, see, e.g. [7,29] and references cited there. In particular the so-called Stolt migration method was successfully applied in [29] to propagate the experimentally measured data closer to the unknown targets". For sampling type methods using dynamic measurements, we refer to the point source method [25], the probe method [8], the linear sampling method [12], the enclosure method [17], the multiple signal classification (MUSIC) method [21] and the total focusing method (TFM) arising from nondestructive evaluation [15]. Note that the TFM is sometimes described as the "gold standard" in the classical beamforming, and it shares the same idea of the so-called Kirchhoff migration widely used in geophysics. Both of them form the image with the superposition of the scattered signals incited by each transducer. They are robust to measurement noise but sensitive to medium noise [1]. In a series of works [1,14], correlation-based imaging schemes have been justified to recover the position of a point reflector in a noisy environment.

The goal of this paper is to show that the TFM and Kirchhoff migration can be used for imaging not only acoustic scatterers with small inclusions but also the shape of regular-size/extended penetrable and impenetrable scatterers with multiple components, including cracks. The proposed method can be regarded as a strengthened version of the total focusing method or Kirchhoff migration in a stationary medium. To our best knowledge, they have been so far applied to the reconstructions of point-like scatterers only. Using synthetic data, in this work we apply for the first time the newly developed strengthened versions of TFM to imaging crack-like and regular-size/extended scatterers with multiple components. In particular, the newly proposed dynamic imaging/indicator functionals show analogous behavior of the time reversal inversion but without having to solve time-dependent wave equations. The promising and salient features of our time-domain sampling method can be summarized as follows. First, the imaging/indicator functionals are formulated directly from the time domain data. No Fourier or Laplace transform is needed on the algorithm level. Second, the imaging schemes do not require the strong *a priori* information of the physical properties of the scatterer, and apply to both penetrable medium and impenetrable obstacle. There is no need to know the type of boundary conditions of an impenetrable obstacle. Third, the imaging schemes are explicit because the imaging indicators do not rely on any matrix inversion or forward solution process. Hence, the method is very robust with respect to measurement noise. Finally, the method is very easy to implement with computational efficiency. Only cheap integrations are involved in calculating the imaging/indicator functionals. In this work, we shall present some mathematical analysis in justifying the indicator behavior of the proposed imaging functionals in the scenario with point-like scatterers. Extensive numerical experiments in two and three dimensions are conducted to demonstrate the feasibility and robustness of our methods. Using the correlation-based algorithms [1,14], we believe that our approach also applies to extended scatterers in a random medium.

The rest of the paper is organized as follows. In the next section we give a brief description of the forward and inverse scattering problems that we are concerned with. Single-source and multi-source indicator/imaging functionals are introduced in Section 3. The behavior of the imaging indicators for point-like scatterers will also be analyzed in this section. Section 4 is devoted to numerical experiments and the paper is concluded in Section 5.

2. Problem setting

Let $D \subset \mathbb{R}^N (N = 2, 3)$ be an open bounded domain with the Lipschitz boundary ∂D and connected exterior $\mathbb{R}^N \backslash \overline{D}$. D represents a stationary scatterer for our study. It is assumed that the exterior $\mathbb{R}^N \backslash \overline{D}$ is an isotropic and homogeneous background medium with a constant phase velocity $c_0 \in \mathbb{R}_+$. Let $\Gamma_i \subset \mathbb{R}^N \backslash \overline{D}$ be a closed Lipschitz surface on which point sources emit incident waves for the probing of the scatterer. Throughout, we assume that the incident wave $u^i_\chi(x, t; y)$ with $x, y \in \mathbb{R}^N$, $t > 0$ is a monopole emitted at the source location $y \in \Gamma_i$ with a temporal pulse signal $\chi : \mathbb{R} \to \mathbb{R}$ such that $\chi(t) \equiv 0$ for $t < 0$. That is, $u^i_\chi(\cdot, \cdot; y)$ is a causal solution to the following wave propagation problem in the free space

$$\Delta u(x, t) - c_0^{-2} \partial_{tt} u(x, t) = -\delta(x - y) \partial_t \chi(t) \quad \text{in } \mathbb{R}^N \times \mathbb{R}^+, \tag{1}$$

$$u(\cdot, 0) = \partial_t u(\cdot, 0) = 0 \qquad \text{in } \mathbb{R}^N, \tag{2}$$