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Immersed boundary method with non-uniform distribution of Lagrangian markers for a non-uniform Eulerian mesh

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ABSTRACT

This study presents a technique to incorporate spheres in a channel flow that uses a non-uniform Eulerian grid using immersed boundary methods with direct forcing. An efficient algorithm is presented which distributes the Lagrangian markers non-uniformly to match the fluid grid and keep the number of markers optimized. Also a novel method to calculate the area weights of the Lagrangian markers is given. It is observed that even the best available algorithms for uniform distribution of markers on a sphere result in a finite error. Using vector spherical harmonics, this error is quantified and reduced to machine precision. A series of simulations of a stationary and moving sphere in a periodic channel at Reynolds number range of 1–100 are presented. Results for a sphere in an ambient shear flow in close proximity of a wall are also shown, where the present non-uniform distribution offers an order of magnitude reduction over uniform distribution of Lagrangian markers. Simulations of a random cluster of 640 monodisperse spherical particles show a 77% reduction in Lagrangian markers with an error of 0.135% in computing the total drag.

1. Introduction

Fluid–structure interaction has been an area of active research over the past few decades. Adaptive meshing and arbitrary Lagrangian Eulerian technique [12] were some of the methods developed to incorporate dynamic or flexible boundaries in a flow. These approaches require expensive remeshing to accommodate the updated position and form of the immersed body. Peskin [23] then introduced the immersed boundary method (IBM) to resolve flow patterns around heart valves. The IBM presents a relatively simple technique to simulate moving immersed bodies. It has proven to be a powerful technique for simulating deformable fluid–solid interfaces as well. Its appeal lies in its ability to use a fixed Eulerian grid, and it does not require a body-fitted mesh. The fluid–solid interface is represented by Lagrangian markers that move with the body. Then a forcing term on these markers is added to the governing equations to account for the boundary conditions on the interface. Mittal and Iaccarino [22] present a review of the various types of IBMs. Their differences lie in the method by which the forcing term is applied. For this study, a direct-forcing method based on Uhlmann [30] is used.

IBM uses a distribution function to interpolate the fluid velocity from an Eulerian grid onto the Lagrangian markers and to spread the forcing term computed at the Lagrangian markers onto the surrounding Eulerian nodes. Every distribution function should ensure the conservation of the forces and torques upon mapping from the Lagrangian markers to the Eulerian grid. One of the more popular distribution functions is known as the Dirac Delta Function (DDF), which was originally proposed for a uniformed spaced Eulerian grid by Roma et al. [25]. The DDF has been designed to conserve total

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force and torque when used with a uniform Eulerian mesh, but the conservation properties do not readily carry over for a non-uniform Eulerian grid. This limitation narrows the capabilities of the IBM in resolving complicated geometries and thin boundary layers, where a non-uniform grid may be desired. As a way around it, several studies, such as Lee et al. [18], utilized a fine uniform mesh in the region that embeds the immersed boundary and a stretched non-uniform grid away of that vicinity. Zhang et al. [36] developed the immersed finite element method that uses the reproducing kernel particle method (RKPM) as a distribution function. This method works for both uniform and non-uniform Eulerian meshes. Later, Pinelli et al. [24] presented a correction to the DDF rendering the IBM functional with both uniform and non-uniform grids.

Our main interest of this study lies in the incorporation of spherical particles in a channel flow code that uses spectral or higher order spatial discretization. This would provide a tool to investigate problems such as riverbed erosions and porous media. Channel flow codes typically use a non-uniform grid in the wall-normal direction for a higher concentration of Eulerian grid near the wall in order to accurately capture the boundary layer dynamics. This non-uniformity of the Eulerian grid can be accommodated in IBM with the use of the correction to the DDF introduced by Pinelli et al. [24].

However, there still exists a significant challenge in terms of the distribution of Lagrangian markers. Most studies in the past have used IBM with an even distribution of Lagrangian markers over the interface. In the case of spheres, a perfectly uniform distribution of Lagrangian markers on the surface of the sphere still remains an open mathematical question, but there are algorithms for obtaining a nearly uniform distribution [26]. For a uniform Eulerian grid, Uhlmann [30] shows that the Lagrangian markers should be spaced such that the surface area associated with each Lagrangian marker must be approximately equal to the square of the Eulerian grid spacing. Any further increase in the number of Lagrangian markers does not lead to any improvement in the representation of the immersed boundary. However, lower number of Lagrangian markers will contribute to under resolution of the immersed boundary as seen by the Eulerian grid on which the flow is resolved. If we follow this criterion for optimal number of Lagrangian markers for a non-uniform grid, the Lagrangian spacing will be dictated by the smallest Eulerian grid spacing surrounding the sphere. For a sphere lying close to the wall, this criterion, along with a uniform distribution of Lagrangian markers will result in an impractical number of Lagrangian markers. In the region close to the wall the spacing between the Lagrangian markers will be comparable to the fine Eulerian mesh and such high resolution is needed to resolve the near-wall flow features. As we move away from the wall, while the Eulerian grid spacing increases, a uniform distribution of Lagrangian markers will correspond to over-resolution, at the expense of significantly enhanced computational cost. To overcome this problem, an algorithm that generates a dynamic non-uniform distribution of Lagrangian markers on a sphere is proposed.

A related problem that this study addresses is the calculation of the Lagrangian volume weights assigned to the Lagrangian markers. These weights are used in the spreading step of IBM which can be expressed as,

$$f(\boldsymbol{x}) = \sum_{l=1}^{N_L} F(\boldsymbol{X_l})\delta(\boldsymbol{x} - \boldsymbol{X_l})\Delta V_l, \tag{1.1}$$

where $F(X_l)$ are the appropriate forcings required at the N_L Lagrangian markers in order to satisfy the desired boundary conditions at the markers. The above expression is used to distribute these Lagrangian forcings to the surrounding Eulerian grid points to obtain $f(x)$, where $\delta(\boldsymbol{x} - \boldsymbol{X_l})$ are the DDF used for spreading. Here ΔV_l are the volumetric weights assigned to each Lagrangian marker point. In the case of uniform distribution of Lagrangian markers, these volume weights are taken to be the same, as a constant. On a non-uniform grid Pinelli et al. [24] suggest a mathematical method to calculate the weights by varying the thickness ε of the interface shell as given by,

$$f(\boldsymbol{x}) = \sum_{l=1}^{N_L} F(\boldsymbol{X_l})\delta(\boldsymbol{x} - \boldsymbol{X_l})\varepsilon_l\Delta A_l, \tag{1.2}$$

where ΔA_l are the geometrical surface areas associated with the Lagrangian markers. The main argument used by Pinelli et al. [24] to calculate the thickness term ε_l is the following. Starting with a Lagrangian function $F(X_l)$, it can be spread onto the Eulerian grid using equation (1.2). Then an Eulerian-to-Lagrangian interpolation follows using,

$$F(\boldsymbol{X_l}) = \sum_{\forall \boldsymbol{x}} f(\boldsymbol{x})\delta(\boldsymbol{x} - \boldsymbol{X_l})\Delta x\Delta y\Delta z. \tag{1.3}$$

With the proper choice of ε_l, one should recover the same value of the initial value of $F(X_l)$ after going through the two steps.

Calculating the geometric surface areas ΔA_l associated with a distribution of Lagrangian markers on a sphere is a challenging task. This study presents a novel numerical technique to calculate the surface areas for any distribution of Lagrangian markers, be it uniform or non-uniform. This new weight allocation reduces the numerical error even in the case of nearly even distribution of markers. It is being presented here in the context of IBM applications, but can be equally useful to numerical modeling in other fields, such as the minimum energy state of electrons on a unit sphere [32,2,13], and arrangement of protein subunits on shells of spherical viruses [15].

The area weights of the Lagrangian markers are evaluated by requiring that the net force and moment on the sphere be computed as accurately as possible. The tractional force on the surface of the sphere is expressed as a multipole expansion of vector spherical harmonics. The appropriate values of the area weights are evaluated by requiring the discrete quadrature