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Adaptive mesh refinement for compressible thermal flow in porous media

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ABSTRACT

We present the implementation of a detailed model for compositional flow in porous media into an adaptive mesh refinement (AMR) framework. This model is applicable to a number of thermal problems. The basic numerical approach is an efficient shock-capturing method designed to minimise the number of expensive phase equilibrium calculations needed per time step. This is implemented in an AMR framework, using a multigrid method for general parabolic equations. The AMR method is applicable to a general class of hyperbolic–parabolic problems which includes the porous media model. We test the method on one-dimensional and two-dimensional thermal gas injection test cases, including non-uniform propagation in a heterogeneous medium.

1. Introduction

Modelling smouldering combustion processes in porous media requires resolving a disparity between the length and time scales of the reaction front, and those of the bulk domain. An example is the in-situ combustion process, which is a thermal recovery method for heavy oils. In this process a combustion front which may be only centimetres in scale is sustained using air injection. The propagation speed of these fronts in the field is very low, only a few centimetres per day. However the typical length scale between wells is large (hundreds of metres), and operating times of wells are long (years). This poses a challenge for simulation methods, which must be able to bridge these scales somehow.

We attempt to bridge the scale gap using an adaptive mesh refinement (AMR) approach. This is well-suited to problems for which the entire domain has some influence on the solution, but only a small subset of the domain needs to be highly resolved. This is the case for problems involving front-propagation in a porous medium, due to the parabolic nature of the pressure equation, and the narrow extent of the front. Some model problems of this type are demonstrated in Section 4. For this kind of problem, the savings available using the AMR approach increase as the ratio of domain size to front extent increases.

The accurate solution of compositional models including stiff reactions requires the resolution of fine spatial and temporal grid scales at the reaction front, which leads to large computational expense. This expense can be reduced by refining the computational grid only where resolution is needed (mainly at the reaction front), by using AMR. The use of AMR for reservoir modelling is complicated by the mixed character (usually hyperbolic–elliptic or hyperbolic–parabolic) of Darcy-type models for low-Reynolds number flow in porous media.

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Table 1
Main physical variables of importance. Quantities typeset in boldface (e.g. $\mathbf{n}$) have dimensions N_C, where N_C is the number of components in the model. Quantities typeset with an over-line (e.g. $\vec{v}_T$) are spatial vectors.

η_α: Phase viscosity [Pa s].	$\mathbf{n}$: Component mole densities [mol cm^{-3}].
c_p^r: Scaled heat capacity of the porous medium at constant pressure [J K^{-1} cm^{-3}].	p: Pressure [Pa].
$\vec{g}$: Gravity [cm s^{-2}].	ϕ: Porosity [between 0 and 1]
H_t: Total enthalpy [J cm^{-3}].	s_α: Phase saturation [between 0 and 1].
K: Absolute permeability of the medium [cm^2].	T: Temperature [K].

Recent work on adaptive meshing for flow in porous media can be found in [1,2]. However, applications in the literature of AMR to reactive simulation and in particular in-situ combustion are quite limited. The work of Christensen et al. [3] looked at AMR for thermal problems including in-situ combustion. They used an implementation of adaptive gridding within the commercial simulator STARS. Greaves et al. [4] also used dynamic gridding within STARS to look at in-situ combustion simulation at lab scale, and in [5] the authors implemented dynamic gridding in Shell's in-house simulator and applied it to an in-situ combustion test.

The paper is structured as follows. In the next section we explain the model and flow equations, and the sequential formulation which is used. In Section 3 we explain the implementation in the AMR hierarchy. We implement a multigrid method for solving elliptic equations on AMR hierarchies and test it on Poisson's equation, including an analysis of the effects of the elliptic flux correction. We consider the extension of this method to a more general class of equations which encompasses the pressure equation of our porous media model, and implement the method within an existing AMR code for the solution of purely hyperbolic problems. In Section 4 we show computational results in one and two dimensions. In this paper we present results without combustion; the development of an in-situ combustion model within this framework will be the subject of a separate paper. Finally in Section 5 we conclude and give directions for further work.

2. Model and sequential formulation

We use the model for thermal, multiphase, multicomponent flow in heterogeneous porous media which was presented in [6]. This model describes the flow of N_C fluid components in an arbitrary mixture of phases; we use a two-phase model with liquid and vapour phases. We define $\mathbf{n}_\alpha$ to be the vector of moles of each component in phase α divided by the pore volume. The quantity $\sum_\alpha \mathbf{n}_\alpha \equiv \mathbf{n}$ is the total component-wise molar density for the combined fluid system. The fluids flow within a porous medium with porosity denoted by ϕ. Table 1 summarises the main physical variables of importance for this model, together with their units.

The flow is governed by equations of mass and energy conservation, Darcy's law, a thermodynamic closure (referred to as the phase equilibrium), heat diffusion, and a reaction system. A sequential formulation leads to a parabolic pressure equation which must be solved together with the main advection–diffusion–reaction system. Full details of the system solved and the single-grid time-stepping algorithm can be found in [7] (non-reactive case), and [8,9] (reactive case); for brevity these are omitted here.

3. AMR discretisation and time-stepping

Although hierarchical adaptive mesh refinement was originally designed for hyperbolic problems, it can also be applied successfully to elliptic problems. The standard approach is to use the AMR grid hierarchy as grid levels in a multigrid algorithm. Particular care must be taken at coarse-fine grid interfaces in order to maintain consistency and convergence of the scheme. This approach can be found in [10–14] among other work, which focused on Poisson's equation and incompressible flow problems.

For flow in porous media, the elliptic or parabolic equation is typically an incompressibility constraint for velocities or, more generally, a pressure equation arising from Darcy's law. Previous work using AMR for flow in porous media focused mainly on incompressible, isothermal Buckley–Leverett type flow, which has an elliptic pressure constraint [1,15,16]. When the fluid is compressible the pressure equation is usually parabolic; this is the case in our own system for flow in porous media. The recent work by Pau et al. [2] extends their previous work and addresses the use of AMR for isothermal compressible systems. This is closely related to our own work, which considers a thermal compressible system with a general phase equilibrium and mass transfer between phases.

A recent overview of work using AMR in computational science can be found in [17]. We follow the classic Berger–Oliger–Colella algorithm [18,19]. AMR uses a multi-level nested hierarchy of rectangular grid patches at different refinement levels. The refinement levels run from level 0, the coarsest, to level l_{max}, the finest. The union of all grid patches of a given refinement level l is referred to as grid level l, and denoted Ω^l. The coarsest grid level Ω^0 covers the entire domain, while successively finer grid levels cover smaller and smaller parts of the domain. The refinement factor between patches on any two given levels is fixed; for this work we use a refinement ratio of 2 between successive levels. Each grid level maintains its own solution, so that in regions where fine patches overlay coarse patches, the coarse data is still present on the coarser grid. Grid levels are required to be properly nested. This means that Ω^{l+1} must be contained entirely within its parent grid