Journal of Computational Physics 262 (2014) 344–357

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A characteristic based volume penalization method for general evolution problems applied to compressible viscous flows ☆

Eric Brown-Dymkoski, Nurlybek Kasimov, Oleg V. Vasilyev *

Department of Mechanical Engineering, University of Colorado Boulder, Boulder, CO 80309, USA

ARTICLE INFO

Article history:
Received 17 May 2013
Received in revised form 18 October 2013
Accepted 31 December 2013
Available online 15 January 2014

Keywords:
CFD
Immersed boundary method
Volume penalization
Compressible flow
Neumann boundary condition
Robin boundary condition

ABSTRACT

In order to introduce solid obstacles into flows, several different methods are used, including volume penalization methods which prescribe appropriate boundary conditions by applying local forcing to the constitutive equations. One well known method is Brinkman penalization, which models solid obstacles as porous media. While it has been adapted for compressible, incompressible, viscous and inviscid flows, it is limited in the types of boundary conditions that it imposes, as are most volume penalization methods. Typically, approaches are limited to Dirichlet boundary conditions. In this paper, Brinkman penalization is extended for generalized Neumann and Robin boundary conditions by introducing hyperbolic penalization terms with characteristics pointing inward on solid obstacles. This Characteristic-Based Volume Penalization (CBVP) method is a comprehensive approach to conditions on immersed boundaries, providing for homogeneous and inhomogeneous Dirichlet, Neumann, and Robin boundary conditions on hyperbolic and parabolic equations. This CBVP method can be used to impose boundary conditions for both integrated and non-integrated variables in a systematic manner that parallels the prescription of exact boundary conditions. Furthermore, the method does not depend upon a physical model, as with porous media approach for Brinkman penalization, and is therefore flexible for various physical regimes and general evolutionary equations. Here, the method is applied to scalar diffusion and to direct numerical simulation of compressible, viscous flows. With the Navier–Stokes equations, both homogeneous and inhomogeneous Neumann boundary conditions are demonstrated through external flow around an adiabatic and heated cylinder. Theoretical and numerical examination shows that the error from penalized Neumann and Robin boundary conditions can be rigorously controlled through an *a priori* penalization parameter η. The error on a transient boundary is found to converge as $O(\eta)$, which is more favorable than the error convergence of the already established Dirichlet boundary condition.

1. Introduction

Numerical simulations of complex geometry flows in a computationally efficient manner, especially for moving surfaces, is a challenging problem. Solid bodies are introduced by imposing appropriate boundary conditions upon surfaces, and to that end, several approaches are used. These methods can be separated into two major groups: body-fitted mesh and

* Corresponding author.
E-mail address: Oleg.Vasilyev@Colorado.edu (O.V. Vasilyev).

http://dx.doi.org/10.1016/j.jcp.2013.12.060

immersed boundary methods. The former uses grids with nodes coincident to the surface of an obstacle, while the latter employs forcing upon the constitutive equations to impose appropriate boundary conditions.

Though body-conformal meshes allow for exact boundary conditions (BCs) to be imposed, the grid must be carefully constructed to precisely fit an obstacle. In most cases, this precludes the use of structured Cartesian grids. The process of mesh generation is highly dependent upon the obstacle geometry, and can become computationally expensive, especially for complex surfaces. This issue is compounded for moving or deforming obstacles, which require continuous adaptation or re-meshing throughout computation of the solution [1].

Immersed boundary methods avoid the cost and complications of body meshing by introducing the effects of obstacles upon the governing equations themselves. Solid body effects, thus embedded within the flow itself, obviate the rigors of positioning nodes upon a surface. Immersed boundary forcing can be applied either to the continuous or discretized equations. While applying discretized forcing allows for a high level of control based upon the numerical accuracy and conservative properties of the discretization method, this approach lacks generality and flexibility across solvers [1].

Volume penalization, on the other hand, imposes the effects of solid bodies by introducing forcing terms on the continuous equations and the resulting evolutionary equations are discretized and solved in the normal manner. One such method is the Brinkman Penalization Method (BPM) [2], which was originally developed for solid, isothermal obstacles in incompressible flows. A principal strength of Brinkman penalization is that error can be rigorously controlled *a priori*, with the solution converging to the exact in a predictable fashion [3,4]. Much work has been done to refine BPM for various numerical techniques, including pseudospectral methods [4–6], wavelets [7,8], and finite-element/finite-volume methods [9]. Of particular note is the impact of volume penalization upon pseudospectral methods, as it has allowed for arbitrary domain geometry and the ability to circumvent the limitations of periodic boundaries [5,6]. In addition to being extended to various solvers, BPM has been expanded beyond the original application of incompressible flows to compressible [7,10] and inviscid [11] regimes.

For all of this progress, boundary conditions imposed by BPM have lacked generality, especially for compressible flows. They have been typically limited to isothermal obstacles and slip/no-slip conditions for the inviscid and viscous regimes, respectively. Additional boundary conditions have been developed on an individual, and usually problem specific, basis. Though homogeneous Neumann condition was recently formulated for scalar mixing and advection–diffusion problems [12], general treatment of homogeneous/inhomogeneous Robin and Neumann conditions has been limited to finite-volume/finite-element methods [9]. In this way, BPM has been inapplicable for many fluid problems, notably those demanding heat-flux and insulating boundary conditions on solid surfaces.

In this work, we propose an extension of volume penalization that introduces characteristic-based forcing terms, exploiting their hyperbolicity to impose general homogeneous and inhomogeneous Neumann and Robin boundary conditions. This Characteristic-Based Volume Penalization (CBVP) method is flexible and can be applied to parabolic and hyperbolic evolutionary equations; in this paper, CBVP is examined for both scalar diffusion and the fully compressible Navier–Stokes equations. As with BPM, this method maintains rigorous control of the error through *a priori* chosen parameters for all boundary conditions.

Characteristic-based volume penalization is well suited for use with adaptive mesh refinement (AMR). As volume penalization does not require body-conformal meshing, high resolution is required around surfaces for computational accuracy and proper definition of geometry. The use of AMR grids maintains solid geometry resolution without over-resolving flow structures. Additionally, the number of nonphysical points lying inside of the obstacle can be minimized to those necessary to support the boundary conditions, which is particularly important for obstacles inhabiting a large portion of the computational domain. All of the results reported in this paper were obtained using the Adaptive Wavelet Collocation Method (AWCM), a general numerical solver which utilizes a wavelet decomposition to dynamically adapt on steep gradients in the solution while retaining a predetermined order of accuracy [13–15]. Employing a rectilinear grid, AWCM precludes the use of body-fitted meshes to impose solid obstacles for all but the most simple geometries. Therefore, volume penalization is a natural means for introducing solid obstacles. As the geometry definitions are treated as any other flow variable by AWCM, and the local grid efficiently and dynamically adapts to resolve surfaces, even for moving obstacles.

2. Characteristic-based volume penalization

2.1. Penalized boundary conditions

Characteristic-Base Volume Penalization imposes Dirichlet, Neumann, and Robin type boundary conditions by introducing forcing terms into the constitutive equations. Consider a domain Ω containing obstacles O_m, and governed by a generalized evolution equation

$$\frac{\partial u}{\partial t} = \mathrm{RHS} \tag{1}$$

outside of O_m, where RHS is simply the physical right hand side forcing terms. Eq. (1) can be hyperbolic or parabolic in nature. A masking function, $\chi(\mathbf{x}, t)$, is defined across Ω, where

$$\chi(\mathbf{x}, t) = \begin{cases} 1 & \text{if } x \in O_m, \\ 0 & \text{otherwise}, \end{cases}$$

separates the domain into a physical region and a penalized region defined by the solid obstacle.