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The systematic design of visual languages applied to logical reasoning ☆

Björn Gottfried

Centre for Computing and Communication Technologies, Universität Bremen, Germany

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ABSTRACT

Visual languages are distinguished by a number of graphical objects and their relations, usually arranged in the two-dimensional plane. While objects and relations are syntactical containers which are used to represent some information, the question arises how to systematically treat all possible syntactical containers given the richness and complexity of the underlying geometry. This paper adopts the intersection paradigm applied in the context of spatial reasoning, which ensures the systematic identification of all conceivable well-formed diagrams. This allows the exhaustive analysis of a visual language. As an example, it is shown how this method enables a thorough understanding of the relations of the graphical elements of linear diagrams which represent monadic first-order logic. The consideration of indeterminate sets even demonstrates the effectiveness of this approach for a representation that includes a total of 512 relations.

1. Introduction

This paper is about the development of visual languages, in particular diagrammatic representations [14,2,9]. As for symbolic representations, issues such as the expressiveness [18] as well as soundness and completeness of diagrammatic formalisms arise [21]. But there are also characteristics which are specific to visual languages, especially a careful choice of graphical elements and their deployment. In the following, the comparison of symbolic and visual representations reveals a fundamental issue for the design of diagrammatic systems, which concerns the systematic analysis of possible relations among the graphical objects of visual languages.

1.1. Symbolic versus visual representations

Symbolic representations use symbols as representation containers, while visual representations employ graphical objects for this purpose. In order to characterise the form of the entities of a representation, well-formedness conditions are defined. These conditions describe permitted combinations of strings in the case of symbolic representations, and correspondingly, permitted combinations of graphical objects in the case of visual languages.

1.1.1. Unambiguous geometric distinctions

Symbolic representations are build upon alphabets whose elements are concatenated to strings and well-formedness conditions can be defined quite easily for strings. Object arrangements of diagrammatic representations are less transparent: the set of well-formed diagrams of a visual language needs to be geometrically unambiguous. This raises the question how it can be guaranteed that each pair of different words of the visual language to be designed has a different geometric representation. The solution to this problem is less straightforward than for strings, given the richness of geometry.

Frequently, visual languages are defined by means of formal grammars, and then, the ambiguity issue concerns

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E-mail address: bg@tzi.de

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the production rules which need to derive for two different words different visual representations. Indeed, the specification of the relationship between visual syntax and semantics is necessary for each visual language [13,21]. However, the ambiguity issue is either neglectfully dealt with through a list of constraints that characterise well-formedness conditions or it is not considered at all. The importance to assure an unambiguous representation is already evident for visual languages which consist only of a small number of primitives, as the compositions of primitives quickly yield complex diagrams [1]. For visual grammars [1,12,8] the visual unambiguousness of the language needs to be verified by iterating over all productions, taking into account any geometric consequences when applying each single production. This is similar to how shape grammars work, which create geometric objects through the successive application of productions [24].

1.1.2. Exhaustive geometric distinctions

Besides the necessity that two different words have different visual representations, there is another issue related to the well-formedness conditions. Strings of symbolic representations usually need to be inspected carefully in order to recognise whether a string is well-formed or contains a mistake; different strings, if well-formed or not, look similar to each other, as they are all defined by the linear concatenation of symbols. By contrast, diagrammatic representations can make use of the full repertoire of geometry, enabling one to find a set of clearly distinguishable object arrangements as information containers. The idea is to avoid correctly looking diagrams that are ill-formed and to exclude distracting configurations that are irrelevant for the representation at hand.

For this purpose, it is necessary to find geometric features which are employed to distinguish the words of a visual language. Those features are to be exhaustively employed in order to represent all well-formed diagrams. Simultaneously, any configurations that are not well-formed but nevertheless defined by the very same kind of geometric features are to be avoided. In this way, all permitted configurations cover all possibilities with respect to the chosen geometric features. Usually, however, conventional techniques for the design of visual languages consist in the decision for a number of specific geometric objects (e.g. circles and arrows) and specific combinations of them (how to connect circles and arrows) [1,4,21]. But such an approach generally results into visual languages which do not clearly separate well-formed diagrams from correctly looking diagrams that are not part of the language.

1.1.3. Exhaustive and unambiguous representations

In a nutshell, there are two issues to be taken into account when designing the visual vocabulary of a visual language: a specific geometric mode of expression should be exhaustively deployed in order to represent all well-formed diagrams, while two different words of the language need to be represented by two different geometric configurations. To put it a different way, each word of the language should be unambiguously represented and each representation container defined by the chosen features should be part of the representation.

Visual grammars implicitly realise an unambiguous representation by defining a set of production rules that derive different visual representations for different words [16]. That the visual representation is simultaneously exhaustive has not been played a role for visual languages yet, since the exhaustive exploitation of a visual mode of expression is formally not necessary for visual languages. However, it helps in the definition of a clear visual layout that assigns to all well looking diagrams a meaning, and conversely, it avoids any well looking diagrams that are not well-formed.

This paper argues that this should be seen as one of the crucial distinctions between visual and symbolic languages. The latter are defined upon strings which frequently look well-formed even though if symbols are in a wrong order. Unfortunately, conventional visual languages suffer from the same drawback, if they are defined by a grammar with a number of shapes as terminal symbols and a number of combination rules which allow specific connections between terminal shapes.

1.2. Spatial reasoning and visual representations

In spatial reasoning [3] the systematic consideration of all object arrangements is also an issue: the spatial state of affairs is to be described for specific objects which are embedded in two-dimensional Euclidean space, on the surface of a sphere, within three-dimensional or any other embedding space. In spatial reasoning a number of relations need to be identified, so that the set of relations are jointly exhaustive and pairwise disjoint, representing some field of interest; undefined relations are to be avoided, so that there are no well looking object arrangements without an associated meaning. It seems therefore worth the effort to look whether approaches in spatial reasoning can be adopted in order to describe exhaustively and uniquely the elements of a visual language.

Such an approach has been investigated since the late 1980s by Max Egenhofer who has been studying *intersection calculi* in order to represent and reason about topological relations [5]. His method allows a systematic and formal identification of possible relations among geometric objects. Depending on the dimensions of the objects and their embedding spaces, the intersections among the interiors of objects, their boundaries, and exteriors can be computed. In particular, the distinction can be made whether those intersections are empty or not. The consequence are a number of topological relations among pairs of objects, for example, among regions [6], lines [20], or arrows [15].

Adopting this approach for the definition of a visual language amounts to characterise all possible object arrangements by means of intersections, that define the aforementioned geometric features. Analysing only those arrangements that are compatible with the well-formedness conditions, allows the unambiguous and exhaustive investigation of the visual language at hand. This approach is applied to a diagrammatic representation in the following sections.