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Flight trajectory design in the presence of contrails: Application of a multiphase mixed-integer optimal control approach

Manuel Soler *, Bo Zou, Mark Hansen

Department of Bioengineering and Aerospace Engineering, Universidad Carlos III, Madrid, Spain
Department of Civil and Materials Engineering, University of Illinois at Chicago, IL, USA
Department of Civil and Environmental Engineering, University California at Berkeley, CA, USA

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ABSTRACT

In this paper we study the 4D trajectory planning problem in a contrail sensitive environment. We identify the control inputs that steer the aircraft from the initial fix to the final fix following a horizontal route of waypoints while performing step climbs and descents, in order to minimize the overall flying cost of fuel consumption, CO_2 emissions, passenger travel time, and persistent contrail formation. The optimal trajectory design problem is formulated as a multiphase mixed integer optimal control problem, which is converted into a mixed integer non-linear program by first making the unknown switching times part of the state, then applying a Hermite–Simpson direct collocation method, and finally introducing binary variables to model the decision making. We solve the mixed-integer nonlinear program using a branch-and-bound algorithm. The numerical results show the effectiveness of the approach.

1. Introduction

Worldwide aviation and the associated greenhouse gas emissions have witnessed significant growth over the past decades. This growing trend is likely to continue in the foreseeable future (GAO, 2009; Owen and Lee, 2006). McCollum et al. (2009) project that the greenhouse gas emission from the aviation sector will increase by 60% and 300% by 2030 and 2050, respectively. The share of emitted CO_2 in the global total is also expected to become more important, from 2% in 1999 to 3–5% in 2050 (Penner et al., 1999). In terms of anthropogenic radiative forcing, an estimate from the United Kingdom (UK) Royal Commission of Environmental Pollution (RCEP) suggests that the aviation sector will be responsible for 6% of the global total by 2050 (Royal Commission, 2002).

The climate change impact of aircraft operations comes from multiple constituents, with carbon dioxide (CO_2) the most known one. Aviation induced NOx also tends to increase tropospheric ozone and reduce methane. However, the increase in radiative forcing associated with ozone is largely offset by the methane reduction, resulting in a relatively small net positive NOx impact compared to the CO_2 impact in aviation operations (Williams et al., 2003). Another important source of aviation-induced climate change that has garnered growing attention is the formation of contrails, which are line-shaped clouds composed of ice particles and formed in the wake of jet aircraft at high altitude where the ambient temperature is very low. The physics of contrail formation is well documented and known as the Schmidt–Appleman criterion (Schimdt, 1941; Appleman, 1953). A more recent review of the conditions for contrail formation from aircraft exhausts can be found in Schumann (1996).

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* Corresponding author at: Department of Bioengineering and Aerospace Engineering, Universidad Carlos III, Madrid, Spain.
E-mail addresses: masolera@ing.uc3m.es (M. Soler), bzou@uic.edu (B. Zou), mhansen@ce.berkeley.edu (M. Hansen).

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Contrails evaporate quickly if the ambient air is dry, but can persist if the ambient air is humid enough. Like natural high clouds, persistent contrails modify the radiation budget of the earth-atmosphere system by reducing the outgoing terrestrial radiation more than they reflect solar radiation, resulting in warming of the earth's surface (Williams et al., 2002). Quantifying the climate impact of persistent contrails has attracted considerable research interests over the past, although consensus has yet to be achieved. The general conclusion is that the magnitude of contrail climate impact is non-negligible compared to that of CO_2 (Penner et al., 1999; Schumann, 2005; Mannstein and Schumann, 2005). At the high end among the existing estimates, the greenhouse effect from aviation induced contrails is approximately 10 times higher than from CO_2 emitted by aircraft (Mannstein and Schumann, 2005). Accounting for the formation of persistent contrails, therefore, is indispensable to mitigating the overall aviation induced climate impact.

Persistent contrail can only be formed when aircraft fly into parts of the airspace in which both the Schmidt–Appleman criterion is met and the atmosphere is sufficient humid.[1] In this paper, we term such airspace as Persistent Contrail Formation Areas, or PCFAs. Mitigating aviation induced contrail formation therefore involves adjustment of flight profiles -both vertically and horizontally- in order to avoid PCFAs. Prevailing approaches for modeling the spatial adjustment of flight trajectories include mathematical programming, simulation, and optimal control. Wei et al. (2012) develops a linear program with binary decision variables for flight level allocation subject to operational feasibility constraints. Also, a linear program with binary decision variables is used by Chen et al. (2012) to consider a tradeoff between contrail formation and emissions in the US airspace. Mixed integer programming techniques are employed in Campbell et al. (2008, 2013), which minimizes aircraft fuel cost while avoiding the formation of persistent contrails. A more recent effort by Zou et al. (2013) formulates a binary integer program that allows for both altitude and heading modifications while minimizing the total flying cost in a dynamic, contrail sensitive environment. Using simulation tools (Gao, 2013; Chen et al., 2014; Sridhar et al., 2013, 2014), model efficient trajectories that minimizes the combined climate impacts of aircraft CO_2 emissions and contrails (and oxides of nitrogen in the latter study), and the tradeoff between the climate impact and aircraft operating cost. Several other attempts have been made to examine vertical displacement of flight paths in order to avoid PCFAs (Williams et al., 2002, 2003; Fichter et al., 2005; Chen et al., 2012). These studies find altitude adjustment to be an effective strategy to significantly reduce contrail production, but with resulting increase in flight separation minima violations, and add workload for air traffic controllers to resolve the trajectory conflicts (Williams et al., 2002; Williams and Noland, 2005).

While using mathematical programming and simulation techniques in aircraft trajectory design allows many constraints (e.g., flight separation minima, maximum workload for air traffic controllers) to be considered, the two approaches have limited capability to model aircraft dynamics. This deficiency can be largely overcome by employing the optimal control approach, which provides control inputs as part of the solution to steer aircraft in the airspace. Previous attempts in using optimal control focus on horizontal design of flight paths with the trade-off between persistent contrail formation and aircraft fuel consumption, under the assumption of constant airspeed and for a range of separate flying altitudes (Sridhar et al., 2011a,b). However, for a given flying altitude, it has been shown that variable airspeed profiles are more efficient than constant airspeed profiles (Pargett and Ardema, 2007; Franco et al., 2010). Allowing aircraft to alter flight altitude is also important in order to permit aircraft maneuvering and thus avoid PCFAs. Joint consideration of variable flying speed and allocation of flight levels remain largely absent in the PCFA-constrained aircraft trajectory literature.

This study contributes to the existing literature by adopting a multiphase mixed-integer optimal control approach, which incorporates both integer and continuous variables into an optimal control problem, to determine the optimal 4-D (time plus 3-D space) aircraft trajectory which allows for varying flying speed and altitude, in the presence of PCFAs. The multiphase mixed-integer optimal control approach has been recently used, but only for horizontal flight paths design with waypoint allocation (Soler et al., 2011; Bonami et al., 2013; Soler, 2013). Altitude change and contrail formations are not considered in those studies. We formalize the research question in the present paper as follows: given an aircraft point mass dynamical model, a route composed of a sequence of horizontal waypoints, and a vertical structure of airspace with multiple permitted flight levels, find the control inputs that steer the aircraft from the initial fix to the final fix following the horizontal waypoints while performing appropriate step climbs/descents, in order to minimize the overall cost of fuel consumption, passenger travel time, and climate impact from CO_2 emission and persistent contrail formation. Binary variables model decision making processes, which herein involve the optimal allocation of flight levels. The times at which both the waypoints and the different flight levels are reached are also obtained within the solution. As part of the modeling efforts, this research presents an approach to estimate the unit costs of fuel consumption, CO_2 emission, and persistent contrail formation.

We solve the multiphase mixed-integer optimal control problem through a multi-step process. First, we reduce the multiphase mixed-integer optimal control problem (MIOCP) to a conventional mixed integer optimal control problem by making the unknown switching times part of the state (Soler et al., 2010). We then apply a collocation method based on the Hermite–Simpson Gauss–Lobatto quadrature rules (Hargraves and Paris, 1987) to convert the conventional mixed integer optimal control problem to a mixed integer non-linear program (MINLP). The MINLP is solved using a branch and bound algorithm to obtain the control inputs and optimal flight trajectory.

The paper continues with a general description of a multiphase MIOCP formulation in Section 2, which is ensued by a proposed solution procedure in Section 3. We devote Section 4 to exposing aircraft dynamics. A detailed case study is presented

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[1] In the remainder of the paper, *contrail formation* always refers to *persistent contrail formation*.