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Evaluation of simulation uncertainty in accident reconstruction via combining Response Surface Methodology and Monte Carlo Method

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ABSTRACT

This paper focuses on the uncertainty of simulation results in accident reconstruction. Since the Monte Carlo Method (MCM) requires a large number of simulation runs, in order to reduce the simulation time of MCM in evaluating the uncertainty of simulation results, a new method named "Response Surface-Monte Carlo Method (RS-MCM)" was proposed. Firstly, Response Surface Methodology (RSM) was used to obtain an approximate model of the true accident simulation model, and then the uncertainty of simulation results was evaluated by combining this approximate model and MCM. The steps of RS-MCM include the generation of sample sets, the determination of response surface model and the statistical analysis of simulation results. The distribution of reconstruction results was obtained using RS-MCM, which can provide more comprehensive information in traffic accident survey, such as the probability of a simulation result at any given confidence interval falling within an arbitrary interval and so on. Finally, three cases have been employed to evaluate the effectiveness of the proposed RS-MCM.

1. Introduction

In accident reconstruction analysis the variable inputs cannot be known with perfect confidence (Untaroiu et al., 2010) . Due to this fact, the question becomes what extent the obtained results for the accident analysis are credible? This question can be tackled with the help of uncertainty analysis techniques (Hong et al., 2013; Remli and Rekik, 2013). There are many methods available for evaluating the overall uncertainty in the result of a set of calculations based on the uncertainty in each input variable. These methods can be divided into deterministic methods and probabilistic methods. Deterministic methods include Total Differential Method (TDM) (Liu, 1999; Williams et al., 2012), Upper and Lower bound Method (ULM) (Lozia and Guzek, 2005; Brach and Brach, 2005), Finite Difference Method (FDM) (Bartlett and Fonda, 2003) and Response Surface Methodology (RSM) (Zou et al., 2012a; Du et al., 2009); probabilistic methods include Gauss Method (Brach and Brach, 2005), Methods with the Use of Stochastic Processes (Lozia and Guzek, 2005), Probabilistic Perturbation Method (Zhang and Li, 2007) and Monte Carlo Method (MCM) (Brach and Dunn, 2004; Kurzhanskiy and Varaiya 2012). The application of these methods in practice can be found in Bartlett (2003), Wood and O'riordain (1994), Bartlett et al. (2002), Li et al. (2004), Fonda (2004), Zou et al. (2012b) and Wach (2007, 2013). And the ULM, FDM and RSM can easily be applied to

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evaluate the uncertainty of simulation results. The range of simulation result to the selected confidence level can be obtained by these three methods, but the distribution of simulation result cannot be obtained.

As a widely used and useful method in uncertainty analysis, the MCM can be used to obtain the distribution of simulation results. However, it requires such a large number of simulation runs that it is impracticable. For example, if PC-Crash (Steffan, 2006) is used to reconstruct an accident and MCM (sample size is 10^6) is used to evaluate uncertainty of results, then the number of simulation runs is 10^6, all these simulation experiments have to be conducted by hand, which is impossible in practice.

Response Surface Methodology (RSM) is a randomized response technique for multivariate modeling and analysis based on Design of Experiments (Jin and Zhang, 2007). It has been widely applied in engineering optimization design (Krajnik et al., 2005; Makadia and Nanavati, 2013) and reliability analysis (Qu, 2004; Bernechea and Arnaldos Viger, 2013) in recent years.

In this paper, a method named "Response Surface-Monte Carlo Method (RS-MCM)" is proposed through combining RSM and MCM, which facilitates uncertainty analysis with MCM using simulation results. Three example cases will be presented.

2. Problem description

In the analysis of pre-accident situations, mathematical models of system man-vehicle-surroundings and data loggers are the two basic groups of procedural methods. This study concentrates on mathematical models of system man-vehicle-surroundings, which include analytical methods and simulation methods.

In this paper, the attention was paid to simulation methods. Models in these methods can be expressed as

$$Y = f(X), \quad X = (X_1, X_2, \ldots, X_s)^T \tag{1}$$

where Y is the accident reconstruction results vector, which commonly represents the pre-impact velocity; X is independent variables vector, e.g., dimension of vehicle; s is the number of independent variables; the function f is an implicit function in simulation methods.

One of the important issues in accident reconstruction is to evaluate the uncertainty of Y caused by the uncertainty associated with X. In this study, RSM is used to obtain a response surface model $Y \approx g(X)$ firstly, and then the uncertainty of Y is evaluated by the combination of MCM and $g(X)$.

3. Response Surface-Monte Carlo Method (RS-MCM)

As mentioned above, the basic idea of RS-MCM is to find an equivalent model $g(X)$ firstly, and then evaluating uncertainty of Y by simultaneously using the $g(X)$ and MCM. It is assumed that the true model of the response may be written as a linear combination of basis functions with some unknown coefficients B. Response surface model can be expressed as

$$Y = f(X) \approx g(X) = Z^T(X)B \tag{2}$$

where $Z(X)$ is the assumed basis function vector, and it can be polynomials with any order or the sum of different basis functions, e.g. sine and cosine functions.

The steps of this method are shown below:

a. Determine distributions of each independent variable.
b. Generate sample sets.
c. Conduct experiments and record experiment results.
d. Determine the response surface model $g(X)$.
e. Evaluating uncertainty of simulation results with MCM.

3.1. Distributions of independent variable

The majority of typical measurements taken by the police officers in accident scenes are assumed to follow normal distributions (Wach and Unarski, 2007). The empirical parameters are usually assumed to follow the uniform distribution if no information is given (Liu, 1999). For example, the friction coefficient between vehicles and dry road is considered to be uniformly distributed in [0.7,0.9]. More information about the distributions of independent variables can be found in Wach and Unarski (2007) and Liu (1999), as to what distribution a specific variable obeys is not discussed in depth in this study, and an assumption is made that distributions of independent variables are known.

3.2. Sample sets generation

Randomized sample sets are commonly generated for both Design of Experiments (DOE) (Qu, 2004; Khuri and Cornell, 1996) and MCM (Olsson and Sandberg, 2002; Myers and Anderson-Cook, 2009) applications. The widely used DOE designs include the orthogonal design (Myers and Anderson-Cook, 2009), central composite design (Qu, 2004) and Uniform Design (UD) (Fang, 1980; Fang and Lin, 2003; Tang and Xu, 2013).