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# Is equilibrium in transport pure Nash, mixed or Stochastic?

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ABSTRACT

The classical theory of transport equilibrium is based on the Wardrop's first principle that describes a Nash User Equilibrium (UE), where in no driver can unilaterally change routes to improve his/her travel times. A growing number of economic laboratory experiments aiming at testing Nash-Wardrop equilibrium have shown that the Pure Strategy Nash Equilibrium (PSNE) is not able to explain the observed strategic choices well. In addition even though Mixed Strategy Nash Equilibrium (MSNE) has been found to fit better the observed aggregate choices, it does not explain the variance in choices well. This study analyses choices made by users in three different experiments involving strategic interactions in endogenous congestion to evaluate equilibrium prediction. We compare the predictions of the PSNE, MSNE and Stochastic User Equilibrium (SUE). In SUE, the observed variations in choices are assumed to be due to perception errors. The study proposes a method to iteratively estimate SUE models on choice data with strategic interactions. Among the three sets of experimental data the SUE approach was found to accurately predict the average choices, as well as the variances in choices. The fact that the SUE model was found to accurately predict variances in choices, suggests its applicability for transport equilibrium models that attempt to evaluate reliability in transportation systems. This finding is fundamental in the effort to determining a behaviourally consistent paradigm to model equilibrium in transport networks. The study also finds that Fechner error which is the inverse of the scale parameter in the SUE model is affected by the group sizes and the complexity of the cost function. In fact, the larger group sizes and complexity of cost functions increased the variability in choices. Finally, from an experimental design standpoint we show that it is not possible to estimate a noise parameter associate to Fechner error in the case when the choices are equally probable.



## 1. Introduction

Transportation planning models have hinged on the classical theories of transport equilibrium that are based on Wardrop's principles (Wardrop, 1952), which is essentially a Nash equilibrium in which all users minimize their travel cost and have no incentives to change their chosen routes. Due to the extensive reliance on transportation planning models for allocation of billions of dollars (euros) of funds on transportation improvements, it has become critical to evaluate theories that underlie these models. Over the past several decades, other transportation equilibrium models such as the Stochastic User Equilibrium model (Daganzo and Sheffi, 1977) and Mixed Strategy Nash Equilibrium model (Bell, 2000) have been

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proposed. Stochastic User Equilibrium is a state of traffic flows where no user may lower their perceived travel time by unilaterally changing their current route, where in the perceived travel time may be associated with random errors. In Mixed Strategy Nash Equilibrium, the user is unable to reduce their expected trip cost by changing their strategies, which are the user's path choice probabilities.

In recent year, several agencies have increased their focus on improving travel time reliability in networks. Variability of link travel time in a network can be attributed to variability in traffic demand (Hanson and Huff, 1988; Pas and Sundar, 1995; Stopher et al., 2008), capacity or supply (Gayah et al., 2013; Kim and Mahmassani, 2014), route choice behavior (Mirchandani and Soroush, 1987; Uchida and Iida, 1993; Jha et al., 1998; Lo and Tung, 2000; Bell, 2000; Yin and Ieda, 2001) and departure time choice behavior (Noland et al., 1998; Noland and Polak, 2002; Coulombel and de Palma, 2014). The variability due to departure time and route choices is associated to coordination errors due to strategic interactions which also contribute to the observed variability in travel time. Therefore, from a modeling standpoint it is important for models to be able to accurately predict variability in travel costs, associated to choice behavior under strategic interactions.

The increased reliance on equilibrium models in transportation to make expensive decisions leads to the fundamental question: Which equilibrium theory is behaviourally consistent? Unfortunately, field data does not provide enough control to test these equilibrium theories. A key feature of these equilibrium theories is the strategic interaction. However, most models estimated using field data (Fosgerau, 2006; Frejinger and Bierlaire, 2007; Carrion and Levinson, 2013; Dixit, 2013) consider travel times as exogenously given, without considering strategic interactions. The strategic interaction results in choices that are a consequence of feedback of the impact of the choice probabilities on path costs, which is how Stochastic User Equilibrium (SUE) is modeled in transport networks. Therefore, it is critical to understand the effects of these strategic interactions on the ultimate choice models that are used in modeling traffic.

Recently, methods from experimental economics are being used to test key theoretical aspects in transportation through controlled laboratory experiments. The key feature of experimental economics is the use of monetary incentives to create controlled environments, specifically these monetary incentives are high enough to induce the utilities and mitigate the impact of an individual's innate characteristics on the utility[1]. In this study we use several transport experiments, where individuals make choices with strategic interactions and their monetary payoffs are affected by endogenous congestion. Therefore strategic interactions and equilibrium play a role in their choices.

Most of these laboratory experiments have implemented various versions of "congestion games" (Rosenthal, 1973), that consists of non-cooperative games in which a player's strategy is to choose a subset of resources, and the utility of each player only depends on the number of players choosing the same or some overlapping strategy. For instance, several studies (Ramadurai and Ukkusuri, 2007; Ziegelmeyer et al., 2008; Daniel et al., 2009) conducted lab experiments to evaluate equilibrium predictions based on a discrete version of a bottleneck model (Vickrey, 1969; Arnott et al., 1993). Differently, Morgan et al. (2009), Rapoport et al. (2006), Hartman (2012) and Denant-Boemont and Hammiche (2012) try to observe within the lab some paradoxes that could be produced by Wardrop-Nash equilibrium when users have to choose among routes (Braess Paradox) or among modes (Pigou-Knight-Downs Paradox, Downs-Thomson Paradox[2]). Anderson et al. (2008) conducted an experimental study on a Market Entry Game (MEG) to represent the traffic coordination problem, with an aim to observe how congestion charges and real-time information given to participants might improve efficiency of the system. In this vein, Denant-Boemont and Fortat (2013) studied the effect of non-linear cost functions, which are similar to the Bureau of Public Roads (BPR) function, on the equilibrium properties, the data from which is used in this study to evaluate the impact of realistic cost functions. In addition, experimental economics has also been used to study the impact of, drivers' risk aversion and subjective beliefs of risk on crash propensity (Dixit et al., 2014) as well as route choice (Dixit et al., 2013). Most of these studies provide strong evidence for a Mixed Strategy Nash Equilibrium (MSNE) in predicting the observed aggregate choices. However, MSNE has been found to underestimate the variances in choices.

An interesting dimension of behavior in experimental games is that even small changes in the parameters of a game, that should not theoretically change the equilibrium, can have dramatic effects on the observed level of efficiency or coordination among players (see Goeree and Holt, 2005). This study attempts to evaluate the performance of different equilibrium theories, particularly: Pure Strategy Nash Equilibrium, Mixed Strategy Nash Equilibrium[3] and Stochastic User Equilibrium.

This study shows that the Pure Strategy Nash (Wardrop) Equilibrium (hereafter called PSNE) is not able to explain the observed choices well. In addition, though Mixed Strategy Nash Equilibrium (hereafter denoted as MSNE) was found to explain better mean choices, however, they do not explain the variance in choices. The Stochastic User Equilibrium (SUE) was found to fit the data the most accurately. Apart from studying the performance of the various equilibrium theories compared to the observed data, this paper presents a method to estimate the choice model for a SUE, and understand the impact of group size as well as the complexity of the cost function on the *scale parameter* (inverse of the Fechner error) of the SUE model.

Our paper is organized as follows. The next section is dedicated to the experimental data. Section 3, provides a more detailed presentation on the method used to estimate the stochastic user equilibrium. Section 4 presents the results, which is followed by the conclusion.

[1] This is a significant feature of experimental economics that was outlined in the seminal work on Induced Value Theory by Smith (1976, 1982)

[2] For a clear review of these paradoxes, see Arnott and Small (1994).

[3] In this paper we only deal with symmetric mixed strategy Nash equilibrium.