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Mellin polar coordinate moment and its affine invariance

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ABSTRACT

The moment-based method is a fundamental approach to the extraction of affine invariants. However, only integer-order traditional moments can be used to construct affine invariants. No invariants can be constructed by moments with an order lower than 2. Consequently, the obtained invariants are sensitive to noise. In this paper, the moment order is generalized from integer to non-integer. However, the moment order cannot simply be generalized from integer to non-integer to achieve affine invariance. The difficulty of this generalization lies in the fact that the angular factor owing to shearing in the affine transform can hardly be eliminated for non-integer order moments. In order to address this problem, the Mellin polar coordinate moment (MPCM) is proposed, which is directly defined by a repeated integral. The angular factor can easily be eliminated by appropriately selecting a repeated integral. A method is provided for constructing affine invariants by means of MPCMs. The traditional affine moment invariants (AMIs) can be derived in terms of the proposed MPCM. Furthermore, affine invariants constructed with real-order (lower than 2) MPCMs can be derived using the proposed method. These invariants may be more robust to noise than AMIs. Several experiments were conducted to evaluate the proposed method performance.

1. Introduction

Images of an object captured from different viewpoints are often subject to perspective distortions [1–4]. If the object is small compared to the camera-to-scene distance, the perspective effect becomes negligible and the affine model provides a reasonable approximation of the projective model [2,5]. Therefore, the extraction of affine invariant features plays an important role in object recognition and registration [6–10]. This method has been used extensively in numerous fields, such as shape recognition and retrieval [11,12], watermarking [13], aircraft identification [14,15], texture classification [16], and image registration [17,18].

The moment-based method is the most commonly used technique for the extraction of affine invariant features. However, only integer-order moments can be used to construct affine invariants [4]. It has been reported that high-order moments are sensitive to noise [19]. Hence, in practice, a moment of the lowest possible order should be used [20]. For similarity transform (including only translation, scaling, and rotation), Fourier Mellin descriptors [21] can be viewed as the invariants constructed by complex number order moments. However, similarity transform is only a special case of affine transform [4]. Thus far, an order of moments for constructing affine invariants can only be an integer. In this paper, we consider generalizing the moment order from integer to non-integer, and construct affine invariants by means of the proposed moment.

1.1. Extraction of features invariant to similarity transform by moment

Similarity transform includes translation, scaling, and rotation [4]. Numerous methods for the extraction of similarity invariants are moment-based [22]. The geometric moment of image $Im(x, y)$ is defined as $gm_{pq} = \int\int x^p y^q Im(x,y)dxdy$, where p, q are non-negative integers, and $p+q$ is known as the order of moment gm_{pq}. Then, the central moment μ_{pq} of the image is defined as

$$\mu_{pq} = \int\int (x-x_0)^p (y-y_0)^q Im(x,y)dxdy, \tag{1}$$

where $x_0 = \frac{gm_{10}}{gm_{00}}, y_0 = \frac{gm_{01}}{gm_{00}}$. We note that the geometric moment order is an integer. In this paper, we refer to the central moment as the traditional moment.

The integer order moment has been used extensively to construct similarity invariants. Hu [23] introduced the moment to pattern recognition for the first time and constructed seven similarity invariants with moments of an order less than 3. Since then, the

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rotational moment [24], complex moment [25], and others, have been proposed and used to construct similarity invariants. It was reported in [19] that high-order moments are sensitive to noise. However, moments used in these methods are all of an integer order. Furthermore, the orders of these moments are no less than 2. As a result, methods based on the traditional moment are noise sensitive.

The Fourier Mellin descriptor, proposed by Sheng et al. [21], is the generalization of the traditional moment. This descriptor is defined as follows:

$$M_{s,l} = \int\int r^{s-1} f(r,\theta) e^{il\theta} dr d\theta, \tag{2}$$

where $f(r, \theta)$ is an image expressed in the polar coordinate system and s denotes a complex number. It was also reported in [21] that Hu's moments are special cases of the Fourier Mellin descriptors provided in Eq. (2). Note that the traditional moment in the polar coordinates system can be expressed as follows:

$$\mu_{pq} = \int\int r^{p+q+1} f(r,\theta) \sin^q\theta \cos^p\theta d\theta. \tag{3}$$

Comparing Eqs. (2) and (3), we note that the exponent of r in Eq. (2) is generalized from an integer $p+q+1$ to a complex number $s-1$. That is, the Fourier Mellin descriptor can be viewed as complex number order moment.

Therefore, moment-based methods have been thoroughly developed for the construction of similarity invariants. A moment of any order (even a complex number order) can be used to construct similarity invariants. However, similarity transform is only a special case of affine transform [4]. We consider the problem of constructing affine invariants by means of non-integer order moments in this paper.

1.2. Problems for construction of affine invariants by moment

1.2.1. Construction of affine invariants based on traditional moment

Affine transform provides a reasonable approximation of the projective model [2]; therefore, affine transform and affine invariants are very important in computer vision. Affine transform consists of a linear transformation, as follows:

$$\begin{cases} \tilde{x} = a_{11}x + a_{12}y + b_1, \\ \tilde{y} = a_{21}x + a_{22}y + b_2. \end{cases} \tag{4}$$

The nonsingular matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ represents the scaling, rotation, and skewing, while $\mathbf{b} = (b_1, b_2)^T$ corresponds to the translation. Similarity transform is simply a special case of affine transform. In fact, when $a_{11} = a_{22}$ and $a_{12} = -a_{21}$, Eq. (4) describes the similarity transform (see [4]).

Affine moment invariants (AMIs) were proposed by Flusser et al. [4,26]. AMIs are the generalization of Hu's moment invariants. Recently, Flusser et al. [27] derived affine invariants by means of graph theory. Using this method, AMIs can be constructed by any integer (no less than 2) order moment. The kernel of these AMIs is defined in terms of the "cross-product" of points (x_1, y_1)*and*(x_2, y_2) in an image [4,27]:

$$C_{12} = x_1 y_2 - x_2 y_1. \tag{5}$$

AMIs have been used in numerous fields, such as image registration [4], object recognition [28], and color image processing [29].

As mentioned previously, high-order moments are sensitive to noise. Therefore, the order of moments used to construct affine invariants should be lower in practice. However, AMIs can only be constructed by integer order moments with methods in [4,26], and the lowest order of moments used for constructing AMIs is 2. As a result, AMIs are more sensitive to noise [30].

1.2.2. Need for modification of moment definition

Numerous moment-based methods have been proposed to improve the robustness of affine invariants to noise. In [31], cross-weighted moments were proposed. Although this method overcomes the sensitivity of the moment to noise to a certain degree, it significantly increases the computation amount. Rahtu et al. proposed a series of algorithms for constructing affine invariants by using properties of the random variable function (see, for example, [30,32,33]). In order to improve the recognition rate, additional parameters should be selected. As a result, the computation is greatly increased. Overall, affine invariants based on traditional moments are sensitive to noise. Certain new methods are computationally expensive.

As mentioned previously, AMIs can only be constructed by integer order moments, and two is the lowest order of moments for constructing AMIs. A natural question arises: can we construct affine invariants by means of a moment with an order lower than 2? The zero-order moment gm_{00} is generally used to normalize other quantities to achieve invariance. One-order moments gm_{01}, gm_{10} are generally used to calculate the centroid in order to achieve translation invariance. Therefore, it is impossible to construct invariants with a traditional moment of an order lower than 2. Consequently, the question changes to the following: can we construct affine invariants with non-integer moments? As mentioned previously, the traditional moment order is an integer. Therefore, we need to modify the moment definition so that we can construct affine invariants with moments of an order lower than 2.

1.2.3. Modification difficulties

In the Cartesian system, the integer power of the "cross-product" must be employed to construct affine invariants in order to lower the computational burden. The power of the "cross-product" C_{12} in Eq. (5) is fundamental in affine invariants (see Eq.(3.12) in [4]). Only an integer power of this "cross-product" can enable AMIs to be calculated by a polynomial of moments. For example, an affine invariant with a second-order moment (without normalization), I_1, can be derived as follows (see [4]):

$$I_1 = \int\int\int\int (x_1y_2 - x_2y_1)^2 \prod_{i=1}^{2} Im(x_i, y_i) dx_1 dy_1 dx_2 dy_2 \tag{6}$$

$$= \int\int\left(\int\int x_1^2 Im(x_1, y_1) dx_1 dy_1\right) y_2^2 Im(x_2, y_2) dx_2 dy_2 + \cdots$$
$$= 2(gm_{20} gm_{02} - gm_{11}^2). \tag{7}$$

We observe from Eq. (6) that I_1 is defined by a quadruple integral. From Eq. (7), we see that I_1 can be calculated by gm_{20}, gm_{02}, and gm_{11}, which are defined by a double integral. Therefore, the computational complexity of AMIs is low. If the power of C_{12} is not an integer (for example, $(C_{12})^2$ is changed to $(C_{12})^{0.5}$ in Eq. (6)), this will result in a quantity that cannot be calculated directly by gm_{pq}. The amount of computation for this is very expensive (see [31]).

In the polar coordinate system, the power for radial factors may differ from the power for the angular part. In the expression of the "cross-product" C_{12}, the radial factors r_1 and r_2 are separated with $\sin(\theta_1 - \theta_2)$:

$$r_1 r_2 sin(\theta_1 - \theta_2). \tag{8}$$

Consequently, the power of the "cross-product" can be generalized into the following form:

$$r_1^{l_1} r_2^{l_2} (\sin(\theta_1 - \theta_2))^{l_3}. \tag{9}$$

Here, it is not necessary for l_3 to be equal to l_1 or l_2. Furthermore, if we employ an integer l_3, the quantity defined in Eq. (9) will result in a polynomial of r_1, r_2, and a certain trigonometric function