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A factor graph evidence combining approach to image defogging

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ABSTRACT

In this paper we introduce an evidence combining inference approach based on factor graphs. The method presented here is quite general in nature and exploits the capability of factor graphs to combine results from multiple algorithms which correspond to different generative models or graphical structures. We do this by using layers across the factor graph to represent each of the algorithms under consideration. For purposes of inference, we convert each of these layers into a simplicial complex using a convex hull algorithm. This allows us to obtain a simplicial spanning tree for each of these simplicial complexes. Making use of this simplicial spanning tree, which corresponds to the reparameterisation of the junction tree of the factor graph, exact inference can be performed using the sum/max-product algorithm. Furthermore, we employ a Procrustean transformation so as to avoid degenerate cases in the inference process. We illustrate how the method can be used for evidence combining in image defogging and compare it against other alternatives elsewhere in literature.

1. Introduction

Many classical problems in computer vision, pattern recognition and data analytics, such as image denoising, dehazing, motion analysis and depth recovery can be viewed as being comprised of multiple interrelated sub-problems with individual constraints. These sub-problems can be considered to be individual systems that are interacting with one another through complex relationships.

Here, we present factor graphs [1,2] as means to modelling such different relationships using a general graphical model, applicable to a wide variety of tasks which can be abstracted as a multilayer graph-based inference problem. The method presented here performs statistical inference by combining sum-products of probabilities that may originate from different graphical structures or generative models. Since factor graphs provide a means for the structural factorisation of a function over several variables, here we define a likelihood function that takes into account the information across layers in the graph. Furthermore, we propose a novel mathematical description to the Markovian factor graph models, which have been shown to be, in theory, exactly solvable using clique trees known as junction trees [3,4].

Similarly, Markov Random Fields (MRFs) [5,6] provide a statistical framework for modelling variables that correspond to adjacent vertices in the graph in a consistent manner. This is the case in many computer vision and image processing applications, where MRFs have found application in image defogging [7–9], image segmentation [10,11] and face recognition and tracking [12]. In these approaches, pixels, edges, corners, etc. are modelled as nodes in a graphical structure and their conditional dependencies are subject to adjacency constraints. In MRFs, each node in the graph is assigned a cost as defined by their unitary relationships [13]. The edges in the graph incident on the node also carry a weight which adds to the cost of assignment. This cost captures the binary relation between adjacent vertices representing the image tokens.

Indeed, coupled, factorial and multilayered Markov random fields (MRFs) can be viewed as different instances of factor graphs. Further, multilayered MRFs have been applied to segmentation [14–16], motion estimation [17,18] and tracking [19]. In these graphical models, each layer corresponds to a belief map derived from different features [14], label fields [15] or scales [16]. For motion analysis, the multiple layers of the MRF correspond to a layer in the graphical model. In a related approach, Ablavsky and Sclaroff [19] have used a layered MRF so as to track partially occluded objects. In [19], the layers are formulated with particular emphasis on representing occluders using mobility and visibility constraints. These “occluder layers” are ordered according to their distance with respect to the camera centre and interact by sharing observations of the tracked object through activity zones.

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Coupled and factorial Markov random fields consider two or more sets of variables which correspond to separate MRFs and model their interconnections between layers more explicitly. In coupled MRFs (CMRFs), the variables are estimated simultaneously through a layer-wise alternating maximisation procedure between layers. These models can be traced back to the work in [20], where the interlinked CMRF consists of one binary valued MRF for both edges and image intensities. Coupled MRFs has been employed for line processes [6,20], image restoration [21] and optical flow [22]. Further, coupled MRFs based on contrasting assumptions can be found in segmentation [23,24] and tracking [25]. In a similar fashion, Narashima [26] performs stereo matching by employing a coupled MRF to model the interactions between object boundaries and their surface normals.

In factorial MRFs [27], the layered structure of the graph is exploited so as to effect inference based upon an alternating expectation-maximisation (EM) algorithm. In [8,28], factorial MRFs are used for defogging/dehazing, where the scene depth map and albedo represent two layers. In this manner, the depth and albedo of the scene are viewed as variables, each described by an independent MRF where the "complementary" variables are given by the observables. In [29,30], closed form solutions for defogging using factorial MRFs have been presented. These make use of a relaxed formulation of the problem in hand [29] or non-linear optimisation [30].

Note that, conceptually, the fundamental difference between coupled MRFs, factorial MRFs and multilayer MRFs is subtle. As mentioned earlier, here we present a method for inference in factor graphs for tasks that can be graphically represented in terms of interconnected variables in a layered arrangement. This treatment naturally leads to *maximum a posteriori* inference using max-product message passing. Thus, this paper not only provides a means for a tractable way to perform inference using local representations, but also allows for factorial, coupled and multilayer MRFs to be viewed as relational structures whose inter and intra layer variable relationships are governed by the factors in the graph.

The rest of this paper is organised as follows. We commence by introducing factor graphs and their reparameterised structure in Section 2, where, in Section 2.1, we show how algebraic topology and homology can be used to describe factor graphs and, in Section 2.2, we ellaborate further on the link between simplicial complexes and spanning trees. In Section 3 we present our inference scheme based upon the max-product algorithm and explain how prototypes can be matched across layers using a Procrustean transformation. In Section 4 we present the step sequence of our method. In Section 5, we illustrate the utility of our method in for image dehazing, which has been traditionally tackled elsewhere using layered MRFs. Finally, we conclude on the work presented here in Section 6.

2. Factor graphs

Our choice of factor graphs stems from the fact that these are a generalisation of probabilistic graphical models, which allow us to formulate the problem in an evidence combining setting. To do this, we note that, in general, graphical models often operate upon observables (pixels, textons, features, etc.) so as to recover variables (labels, values, states, etc.) which can be expressed as a mixture of prototypes. In this manner, available data, *i.e.* observables, can be described making use of a finite set of characteristic or typical examples, *i.e.* prototypes, of the observables under consideration. It is worth noting in passing that the concepts of prototypes and observables above, and used throughout the paper, are consistent with those widely used in the pattern recognition and machine learning literature [31]. This treatment allows for the use of the max-product algorithm in factor graphs [1] for purposes of inference. Factor graphs generalise both Markov random fields and directed graphs such as Bayesian networks. They also model variable relationships that are not necessarily probabilistic, a characteristic which has made them useful in coding theory, error correcting codes [2,32] and signal processing [33,34].

The left-hand panel of Fig. 1 shows a factor graph arising from a 2D lattice. These are typical of computer vision problems. Exact inference for such a graph can be effected by creating a chordal graph through a variable elimination sequence and constructing a junction tree. The chordal graph and junction tree resulting from these procedures for the grid-graph are displayed in the middle and right-hand panel of the Fig. 1. Mathematically, the factor graph shows a factorisation of the joint probability distribution $P(\mathbf{x})$ according to the Gibbs measure, *i.e.*

$$P(\mathbf{x}) = \frac{1}{Z}\prod_{c\in\mathcal{C}}\psi(\mathbf{x}_c), \tag{1}$$

where $\psi(\mathbf{x}_c)$ is a potential function defined over subsets of variables $\mathbf{x}_c$ in the clique c and Z is a normalisation constant.

When the graph is converted into a chordal one, the probability in Eq. (1) can be expressed as follows

$$P(\mathbf{x}) = \frac{\prod_{c\in\mathcal{C}} P(\mathbf{x}_c)}{\prod_{s\in\mathcal{S}} P(\mathbf{x}_s)}, \tag{2}$$

where $P(\mathbf{x}_c)$ is the joint probability of the clique with the subset of variables $\mathbf{x}_c \in \mathbf{x}$ and $P(\mathbf{x}_s)$ is the joint probability of the variables in the separator set $\mathbf{x}_s = \mathbf{x}_{c_i} \cap \mathbf{x}_{c_j}$, where cliques c_i and c_j are adjacent, *i.e.* connected by an edge. As such, the number of separator sets is equal to the number of edges in the junction tree. In Appendix A, we show how the Gibbs distribution in Eq. (1) can be reparameterised so as to arrive at the expression in Eq. (2). Note that Koller et al. [35] also provide a proof for Eq. (2) which is based on message passing. We have included the proof in the appendix since it does not depend on the message passing operation over the graph and, hence, it shows that the factorisation above is independent of the traversal operation across the graph. Later on, we will present a method based on message passing so as to perform inference on the graph.

For the junction tree in the right-hand panel of Fig. 1, the joint probability can be computed making use of the factorised quotient (2).[1] This is important since it opens-up the possibility of approximating the distribution $P(\mathbf{x})$ by obtaining a tree via variable elimination across the factor graph.

2.1. Simplicial complexes

If we view the graphical model in the left-most panel of Fig. 2 as a set of triangles organised in a sequence as obtained from a convex hull or Delaunay triangulation, we can obtain the graph in the middle of Fig. 2. Note that this corresponds to the reparameterisation of the Gibbs distribution in Eq. (1) in terms of *2-simplexes,*

[1] For the junction tree on the right-most panel of Fig. 1, and using Eq. (2) we get

$$\begin{aligned} P(\mathbf{x}) &= \frac{P(x_1,x_2,x_4)P(x_2,x_4,x_5,x_6)P(x_2,x_3,x_6)}{P(x_2,x_4)P(x_2,x_6)P(x_4,x_5,x_6)} \\ &\quad\times \frac{P(x_4,x_5,x_6,x_8)P(x_4,x_7,x_8)P(x_6,x_8,x_9)}{P(x_4,x_8)P(x_6,x_8)} \\ &= P(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9). \end{aligned} \tag{3}$$