Pattern Recognition 78 (2018) 91–102

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/patcog

Disconnectedness: A new moment invariant for multi-component shapes

Joviša Žunić [a], Paul L. Rosin [b], Vladimir Ilić [c,*]

[a] Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia
[b] School of Computer Science, Cardiff University, Cardiff CF24 3AA, Wales, U.K.
[c] Faculty of Technical Sciences, University of Novi Sad, Novi Sad, 21000, Serbia

ARTICLE INFO

Article history:
Received 25 May 2017
Revised 19 December 2017
Accepted 14 January 2018
Available online 17 January 2018

Keywords:
Shape
Multi-component shapes
Moments
Moment invariants

ABSTRACT

In this paper we further develop the recent concept of multi-component shapes, which is applicable to image processing and image analysis tasks. The domain of multi-component shapes is very diverse and includes shapes that correspond to a group of objects that act together (e.g. a fish shoal), natural components of a segmented object (e.g. cells in embryonic tissues), a set of shapes corresponding to the same object appearing at different times (e.g. human gait in an image sequence), and many more.

So far, there are few methods for numerically evaluating multi-component shapes. In this paper we introduce one such method: a disconnectedness measure, that naturally corresponds to multi-component shapes, and has no analogue in single-component shape measures. The new measure depends on the number of shape components, the whole shape but also the shape of its components, on the relative size of the shape's components and their mutual position. All these are natural requirements for a "disconnectedness" multi-component shape measure. In addition, the new measure is invariant with respect to translation, rotation and scaling transformations. The measure is simple and fast to compute.

The disconnectedness measure introduced here is a generic image analysis tool. It has not been developed for a specific application. As such, it can be applied to a variety of applications. Several of them are provided in the paper, as well as synthetic examples that support a better understanding of the behavior of the new measure.

1. Introduction

Shape analysis is a perennial topic in computer vision, and the topic of many books [1–7]. Shape based tools for image analysis have a wide spectrum of applications: astronomy [8,9], medicine [10], ecology [11], botany [12], agriculture [13], archaeology [14], transport [15], particle analysis [16], technology [17,18], just to mention a few. This is because shape has a high discriminative capacity, and shape properties can be evaluated numerically. Different approaches have been applied to characterize shapes numerically. Some of them are generic ones [19–22], aimed to satisfy some specific properties (like rotational [19,22] or affine [21,23] invariance, for example). There are also approaches designed to measure specific shape properties. Shape convexity [24–26], circularity [27,28], squareness [29], tortuosity [30], ellipticity [8,31–33], are examples of shape properties which have been studied and numerically evaluated so far. Some methods for measuring shape properties are quite old [32], while others are recent [34]. Also, a range of different methodologies have been applied: algebraic [19], geometric [35], logical [36], fractal [37], and so on.

In this paper we further develop the fairly new concept of multi-component shapes [38,39]. The concept differs essentially from the existing ones, even if it might be understood as a natural one. Multi-component (i.e. compound) shape is a very generic term. It may relate to: **(i)** Shapes corresponding to a group of the objects that act together; for example, a fish shoal, in which each single fish shape is a component of a multi-component fish-shoal shape (see Fig. 1(a)). **(ii)** Shapes corresponding to an object partitioned on a natural way; an example could be a hand, whose components are the fingers and palm (Fig. 1(b)). **(iii)** Shapes partitioned by criteria not directly or easily visible from an image; an example is a region (e.g. state, country, continent, etc.) divided according to some administrative criteria [40] (Fig. 1(c)). **(iv)** Shapes partitioned with indistinct boundaries, but which still produce relatively naturally recognizable components (Fig. 1 (d)). **(v)** Shapes whose components are the shapes of the same object appearing on consecutive sequence of frames (Fig. 1(e)). Of course, there are

* Corresponding author.
E-mail addresses: jovisa_zunic@mi.sanu.ac.rs (J. Žunić), Paul.Rosin@cs.cf.ac.uk (P.L. Rosin), vlada.mzsvi@uns.ac.rs (V. Ilić).

https://doi.org/10.1016/j.patcog.2018.01.010

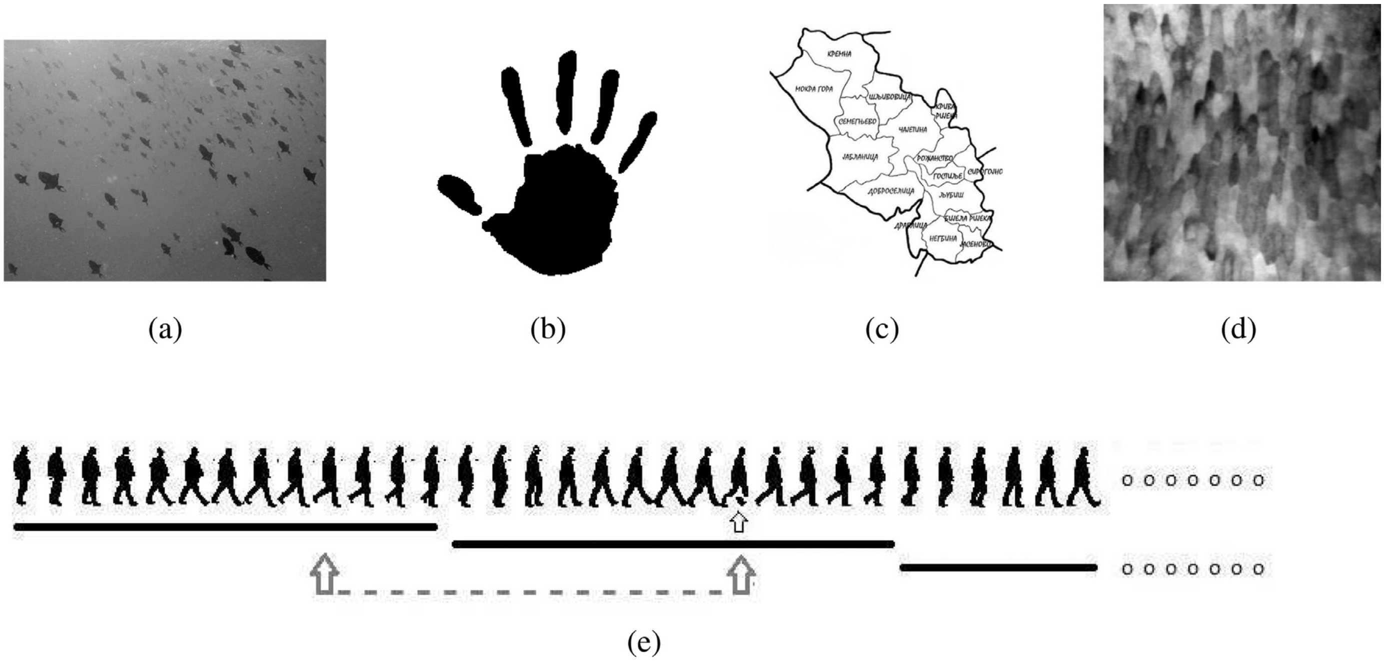

Fig. 1. (a) Fish shoal (b) Palm-print (c) Zlatibor-region partitioned by the village districts (d) Embryonic tissue with indistinct cell boundaries (e) Human gait – considered as a 13-component shape, whose components are the appearances of a walking person in a sequence of 13 consecutive frames.

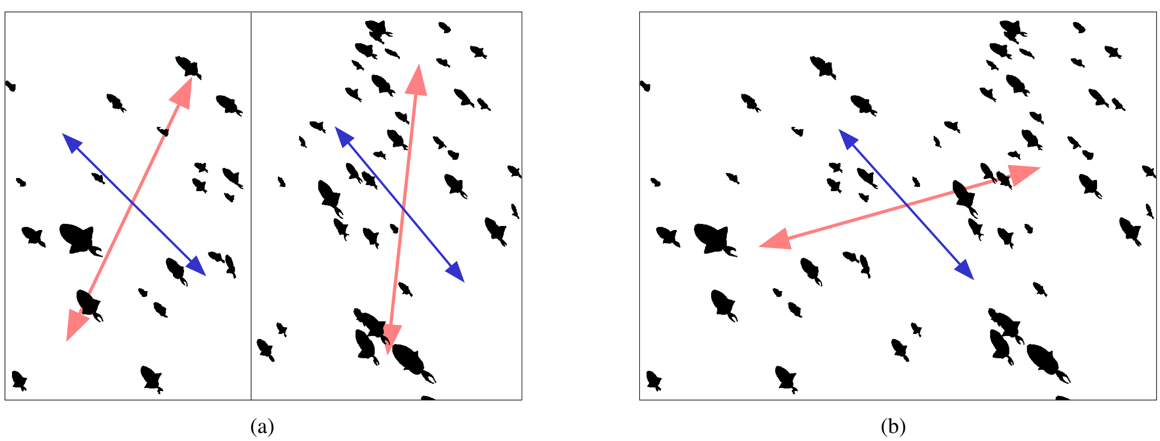

Fig. 2. Orientations computed for (a) the separate left and right halves of the image, and also for (b) the complete image. Shorter dark blue arrows correspond to the shape orientations computed for the three sets of data according to our multi-component approach. In comparison, the long light red arrows correspond to the traditional method for computing orientation, in which each of the three sets of data are considered as representing single-component shapes (each containing multiple fish). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

many more examples. Some of them can be found in Section 4, related to the experiments.

Being a conceptually new approach, the multi-component shape approach has specific demands. Indeed, even a basic shape feature, such as the shape orientation of multi-component shapes, has additional requirements that do not appear when working in the domain of single-component shapes. For example, in the case of multi-component shapes that consist of a huge number of components (e.g. as a fish-shoal does), the computed orientation of such a multi-component shape should be not depend on what portion of the multi-component shape has been captured in the image frame.

In the example of the image in Fig. 2, multi-component shape orientation (represented by shorter dark blue arrows) is computed for both the whole image (Fig. 2b) and also separately for its left and right halves (Fig. 2a). It can be seen that these three computed orientations coincide, which is the preferred outcome, and suggests that the multi-component orientation exists and is an inherent property of the given fish-shoal shape. If the whole shape and its halves are treated as single component shapes (such that all the black pixels in that portion of the image belong to the shape), then the orientations computed (represented by long light red arrows) differ substantially, as these represent the global pattern rather than its contents; for more details see [39].

In this paper we introduce a new shape measure for multi-component shapes. It is named the *disconnectedness of multi-component shapes* or, for short, just the *disconnectedness measure*. The name comes from an intuitive interpretation of what the properties of a disconnectedness measure (of multi-component shapes) should satisfy. It ranges through the interval $[0, \infty)$ and returns the smallest possible value (equal to zero) for single component connected shapes. The new measure depends on the number of components, their mutual position (including the mutual distances among them) and relative sizes, the shape of the components and