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Non-linear thermal simulation at system level: Compact modelling and experimental validation

Mirko Bernardoni[a,*], Nicola Delmonte[b], Diego Chiozzi[b], Paolo Cova[b]

[a] KAI Kompetenzzentrum Automobil- und Industrieelektronik GmbH, Austria
[b] University of Parma, Dipartimento di Ingegneria dell'Informazione, Italy

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ABSTRACT

In this work, a general methodology to extract compact, non-linear transient thermal models of complex thermal systems is presented and validated. The focus of the work is to show a robust method to develop compact and accurate non-linear thermal models in the general case of systems with multiple heat sources. A real example of such a system is manufactured and its thermal behaviour is analyzed by means of Infra-Red thermography measurements. A transient, non-linear Finite-Element-Method based model is therefore built and tuned on the measured thermal responses. From this model, the transient thermal responses of the system are calculated in the locations of interest. From these transient responses, non-linear compact transient thermal models are derived. These models are based on Foster network topology and they can capture the effect of thermal non-linearities present in any real thermal system, accounting for mutual interaction between different power sources. The followed methodology is described, verification of the model against measurements is performed and limitations of the approach are therefore discussed. The developed methodology shows that it is possible to capture strongly non-linear effects in multiple-heat source systems with very good accuracy, enabling fast and accurate thermal simulations in electrical solvers.

1. Introduction

Many compact thermal modelling methods, which use RC networks to describe heat propagation for certain boundary conditions, can be found in the literature.

For instance, Szekely has been focusing on infinite *RC* transmission lines, Network Identification via Deconvolution (NID) [1,2], the concept of structure function in electronic packages [3]; the work carried out in the framework of the DELPHI project, aimed at the determination of Boundary-Condition-Independent (BCI) compact thermal models of several packages used in electronic industry, see [4,5]. Schweitzer [6] showed several methods about how to determine the parameters of a thermal network with *a priori* defined topology; Model-Order-Reduction techniques represent an efficient way to reduce model complexity and such an approach can be found in [7].

Lumped Element (LE) models can be discerned in *physical models*, strictly connected to the physical layers and features of the described systems [8], and *empirical models*, which aim at capturing a given response of the studied system [9,10]. Lumped element physical models tend to be cumbersome to be built, losing their appeal in terms of computational lightness. Therefore, it makes sense to invest effort in developing accurate *behavioural* models. In the field of compact thermal models, a general approach to their determination can be found in [11], while examples of their applications to electro-thermally coupled simulations or coupling with different dynamics can be found in [12–17].

An example of how to insert lumped element models in an FEM model is shown in [18].

A general methodology which allows to determine lumped element models should produce accurate, fast, non-linear models; the presence of multiple heat sources should also be considered. Finite Element Method (FEM) is currently the simulation tool which offers most of the required features, at the cost of simulation speed [19,20]; on the other hand, standard lumped element models offer the best in terms of simulation speed, but they may easily lack in terms of description of non-linearities and accuracy.

In this work, a robust procedure to generate such compact models fullfilling all the above-mentioned requirements is described in detail, together with its validation on an *ad-hoc* test structure.

2. Determination of Foster networks

In this section, a method to obtain a non-linear Foster network from

* Corresponding author at: KAI GmbH, Europastrasse 8, Villach 9524, Austria
E-mail address: mirko.bernardoni@k-ai.at (M. Bernardoni).

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a set of thermal impedance curves obtained at different power dissipation levels is shown.

2.1. Linear Foster networks

The thermal impedance response $Z_{th}(t)$ of a given system is usually described by a series of K exponential terms:

$$Z_F(t) = \sum_{k=1}^{K} R_k(1 - \exp(-t/\tau_k)) \tag{1}$$

with obvious meaning of the symbols. To determine all the parameters, a logarithmically-spaced set of time constants τ_k between two reasonable extremes is generated [21], followed by the minimization of the difference between the measured response $Z_F(t)$ and the calculated one:

$$\min \sum_{t=t_0}^{t=t_s} (Z_F(t) - Z_{th}(t))^2 \tag{2}$$

where $t_0,\ldots,t_s$ are the time instants at which both waveforms are sampled. The fitting parameters in Eq. (2) are the number of stages K and each of the R_k resistances. The minimization can be performed iteratively by increasing the number of stages until a satisfactory fit is achieved, with the lowest number of stages K as possible. Such algorithm can be easily implemented in Python [22] by using the NumPy [23] and SciPy [24] modules.

2.2. Non-linear Foster networks

A non-linear Foster network can be thought as a merging of several Foster networks, each of which has been described like in Eq. (1).

In case of a non-linear system, different power dissipation levels P_0, P_1 will lead to different responses:

$$Z_{th0}(t) = \frac{\Delta T(t)}{P_0} = \sum_{k=1}^{K_0} R_{k0}(1 - \exp(-t/\tau_{k0})) \tag{3}$$

$$Z_{th1}(t) = \frac{\Delta T(t)}{P_1} = \sum_{k=1}^{K_1} R_{k1}(1 - \exp(-t/\tau_{k1})) \tag{4}$$

with $K_0 \neq K_1$ in general, as well as the R_{k0}, R_{k1} and τ_{k0}, τ_{k1} constants.

It is always possible to find a value $K \geq \max(K_0,K_1)$, for which the two responses are both equally well described, both with the *same number of terms*. Calling Q the number of power dissipation levels, this observation can be easily generalized to $Q > 2$.

Therefore, if the topology of the network is fixed i.e., the amount of stages K needed is the same for every power dissipation level, it is possible to collapse all the linear Foster networks into a *single, non-linear* Foster network. The terms $R_1,R_2,\ldots,R_K$ are dependent on the temperature of the network's first node (which is the only node with a physical meaning).

The topology is therefore described in Fig. 1 where only three stages are drawn, for the sake of simplicity.

Accurate non-linear models can be built only with non-linear resistive terms, keeping the capacitive terms constant [25].

Moreover, numerical problems are reduced if the variation of each of the terms $R_1(T)$, $R_2(T)$, …, $R_k(T)$ is monotonic. It was noted that the best number of stages which satisfies the listed requirements was characterized by a (not strictly) monotonic behaviour of the non-linear, resistive terms as a function of the input temperature. Should this not be the case, it is sufficient to increase the number of stages K.

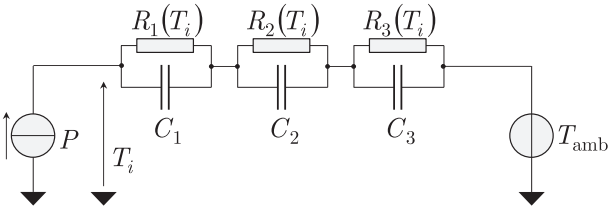

Fig. 1. An exemplary non-linear Foster network with three stages.

The procedure to determine the non-linear thermal models here shown consists of the following steps:

1. Generate a set of Z_{th} curves for increasing *power dissipation* levels P_1, …,P_Q with $P_1 < P_2 < \cdots < P_Q$;
2. Starting from the lowest power level P_q with $q = 1$, perform the standard procedure to obtain a linear Foster network which describes this very thermal impedance; an initial number of stages K will be obtained;
3. Go to the next power level P_{q+1} and perform again the standard procedure to obtain a linear Foster network, using the coefficients obtained at the previous step P_q as starting guess points for the optimization; at the end of this step, q Foster networks, each of which is made of K stages, are obtained;
4. Check the series of values obtained for $R_1,\ldots,R_K$; if the fitting is consistent for each power level and the monotonicity of each resistive terms is respected, the procedure is successful. Otherwise, repeat from step 2) with $K = K + 1$.

The monotonicity of the resistive terms can be obtained by careful selection of the time constants which are used to perform the fit. By finely increasing the P_q values, the typical time constants will also vary slightly and the monotonicity of the resistive terms can be easily achieved.

Basically, the correct Foster network is the one for which the following equations hold:

$$Z_h(t) = \sum_{k=1}^{K} R_k(T)(1 - e^{-t/\tau_k(T)}) \tag{5}$$

$$\begin{aligned} &\frac{dR_k(T)}{dT} \geq 0 \quad \text{or} \quad \frac{dR_k(T)}{dT} \leq 0 \qquad \forall\, k \in [1, K] \\ &\frac{dC_k(T)}{dT} = 0 \qquad\qquad\qquad\qquad\quad \forall\, k \in [1, K] \end{aligned} \tag{6}$$

A graphical illustration of the process is described in Fig. 2. The resistance of the k-th stage, calculated in case of a power dissipation P_q is defined as $R_k(T_q)$, where T_q is the temperature obtained at the input node of the given Foster network when a step with amplitude P_q is applied. For higher power dissipation values, different sets of resistive terms are obtained. Three cases of increasing power levels P_1, P_2 and P_3 are shown as an example in Fig. 2 (*a*), (*b*) and (*c*), respectively. The merging of these three linear networks results in a non-linear Foster network, where each resistance non-linearity is defined as a Look-Up-Table (LUT). For instance, referring to the first stage, the LUT is defined as: $(T_1,R_1(T_1))$, $(T_2,R_1(T_2))$, $(T_3,R_1(T_3))$ which results in the network depicted in Fig. 2 (*d*). It is possible to extend this procedure in order to include the effect of different ambient temperatures, resulting in a network where each resistive term is described by a double-entry LUT (Fig. 2 (*e*)).

3. Matrix description of thermal systems

The most general representation of a system is in its MIMO (Multiple Inputs, Multiple Outputs) form. A system with N power sources and M different locations where the temperature is monitored (see for instance [26] and [27]) can be described by an $(M \times N)$ matrix definition as follows:

$$\begin{pmatrix} \Delta T_1 \\ \vdots \\ \Delta T_M \end{pmatrix} = \begin{pmatrix} Z_{11} & \ldots & Z_{1N} \\ \vdots & Z_{mn} & \vdots \\ Z_{M1} & \ldots & Z_{MN} \end{pmatrix} \cdot \begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} \tag{7}$$

where the indexes $m = 1,\ldots,M$ and $n = 1,\ldots,N$, respectively. Such