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Brief paper

Subsystem identification of multivariable feedback and feedforward systems ☆

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ABSTRACT

We present a frequency-domain technique for identifying multivariable feedback and feedforward subsystems that are interconnected with a known subsystem. This subsystem identification algorithm uses closed-loop input–output data, but no other system signals are assumed to be measured. In particular, neither the feedback signal nor the outputs of the unknown subsystems are assumed to be measured. We use a candidate-pool approach to identify the feedback and feedforward transfer function matrices, while guaranteeing asymptotic stability of the identified closed-loop transfer function matrix. The main analytic result shows that if the data noise is sufficiently small and the candidate pool is sufficiently dense, then the parameters of the identified feedback and feedforward transfer function matrices are arbitrarily close to the true parameters.

1. Introduction

Subsystem identification (SSID) is the process of building empirical models of unknown dynamic subsystems, which are interconnected with known dynamic subsystems. These connections can be series, parallel, or feedback. SSID relies on measured data to identify the unknown subsystems. However, not all input and output signals to the unknown subsystems are necessarily accessible, that is, available for measurement.

This paper is concerned with closed-loop SSID of unknown feedback and feedforward subsystems interconnected with a known subsystem as shown in Fig. 1. The exogenous input r and closed-loop output y are measured, whereas internal signals u and v are not assumed to be accessible. We note that closed-loop SSID is distinct from the well-studied problem of system identification in closed loop (Forssell & Ljung, 1999; Isermann & Münchhof, 2011; Van den Hof, 1998; Van den Hof & Schrama, 1995). Specifically, in SSID, the unknown subsystems have inputs or outputs that are inaccessible.

SSID has applications in biology and physics as well as human-in-the-loop systems. For example, many biological systems are modeled by the interconnection of subsystems, which may be unknown and have inaccessible inputs and outputs (Roth, Sponberg, & Cowan, 2014). Similarly, physical systems are often modeled by a composition of subsystems, which are based on either physical laws or empirical information. For example, in D'Amato, Ridley, and Bernstein (2011), a large-scale physics-based model of the global ionosphere–thermosphere is improved by using measured data to estimate thermal conductivity, which can be regarded as an unknown feedback subsystem. In this application, the output of the unknown subsystem is inaccessible.

SSID also has application to modeling human behavior. For example, there is interest in modeling human-in-the-loop behavior for applications such as aircraft (Itoh & Suzuki, 2005; Nieuwenhuizen, Beykirch, Mulder, & Bülthoff, 2007; Nieuwenhuizen & Bülthoff, 2013; Olivari, Nieuwenhuizen, Venrooij, Bülthoff, & Pollini, 2012) and automobiles (Hellstrom & Jankovic, 2015; Macadam, 2003; Steen, Damveld, Happee, van Paassen, & Mulder, 2011). In addition, SSID methods can be used to model human behavior in motor control experiments, which study human learning (Drop, Pool, Damveld, van Paassen, & Mulder, 2013; Kiemel, Zhang, & Jeka, 2011; Laurense, Pool, Damveld, van Paassen, & Mulder, 2015; Zhang & Hoagg, 2016).

Closed-loop SSID of feedback and feedforward models is considered in D'Amato et al. (2011), Gillijns and De Moor (2006), Morozov et al. (2011) and Palanthandalam-Madapusi, Gillijns, De Moor, and Bernstein (2006). However, the identified feedback and feedforward models obtained from the methods in D'Amato et al. (2011), Gillijns and De Moor (2006), Morozov et al. (2011)

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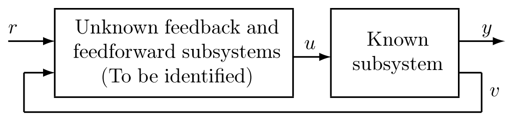

Fig. 1. The unknown feedback and feedforward subsystems are to be identified using the measured data r and y. The internal signals u and v are inaccessible.

and Palanthandalam-Madapusi et al. (2006) can result in unstable closed-loop dynamics. To address closed-loop stability, Zhang and Hoagg (2016) present an SSID technique that guarantees asymptotic stability of the identified closed-loop transfer function. The approach in Zhang and Hoagg (2016) applies to single-input single-output (SISO) subsystems and requires that the measured closed-loop output y is the same as the feedback v.

The new contribution of this paper is a closed-loop SSID method that: (i) identifies multi-input multi-output (MIMO) feedback and feedforward subsystems; (ii) allows for a measured output y that is not necessarily the same as the feedback v; and (iii) guarantees asymptotic stability of the identified closed-loop transfer function matrix. This paper adopts techniques from Zhang and Hoagg (2016) but goes beyond the previous work by addressing MIMO subsystems and allowing for the measured output y to differ from the feedback v. Furthermore, the discrete-time SSID approach in this paper can improve computational efficiency relative to the continuous-time approaches in Zhang and Hoagg (2016). In this paper, the feedforward subsystem model is parameterized as a finite impulse response (FIR) transfer function matrix, which can improve computational efficiency as discussed in Section 7. To accomplish (i)–(iii), we use a candidate-pool approach. Our main analytic result shows that if the data noise is sufficiently small and the candidate pool is sufficiently dense, then the parameters of the identified feedback and feedforward transfer function matrices are arbitrarily close to the true parameters.

2. Notation

Let $\mathbb{F}$ be either $\mathbb{R}$ or $\mathbb{C}$. Then, $x_{(i)}$ denotes the ith component of $x \in \mathbb{F}^n$, and $A_{(i,j)}$ denotes the (i,j) entry of $A \in \mathbb{F}^{m\times n}$. Let $\|\cdot\|$ be a norm on $\mathbb{F}^{m\times n}$, and let $\|\cdot\|_2$ be the two-norm on $\mathbb{F}^n$. Next, let A^* denote the complex conjugate transpose of $A \in \mathbb{F}^{m\times n}$, and define $\|A\|_{\mathrm{F}} \triangleq \sqrt{\operatorname{tr} A^*A}$, which is the Frobenius norm of $A \in \mathbb{F}^{m\times n}$. Let A^{A} denote the adjugate of $A \in \mathbb{F}^{m\times n}$.

Let vec A be the vector in $\mathbb{F}^{mn}$ formed by stacking the columns of $A \in \mathbb{F}^{m\times n}$. Let vec^{-1} be the inverse vec operator, that is, $\operatorname{vec}^{-1}(\operatorname{vec} A) = A$. Let $A \otimes B$ denote the Kronecker product of $A \in \mathbb{F}^{m\times n}$ and $B \in \mathbb{F}^{k\times l}$.

Let $\mathbb{R}[z]$ denote the set of polynomials with coefficients in $\mathbb{R}$, and let $\mathbb{R}^{m\times n}[z]$ denote the set of $m\times n$ polynomial matrices, that is, the set of matrix functions $P: \mathbb{C} \to \mathbb{C}^{m\times n}$ whose entries are elements in $\mathbb{R}[z]$. The degree of the polynomial $p \in \mathbb{R}[z]$ is denoted by deg p, and the degree of the polynomial matrix $P \in \mathbb{R}^{m\times n}[z]$ is denoted by $\deg P \triangleq \max_{i=1,\ldots,m;j=1,\ldots,n} \deg P_{(i,j)}$.

Define the *open ball of radius* $\epsilon > 0$ *centered at* $c \in \mathbb{F}^{m\times n}$ by $\mathbb{B}_\epsilon(c) \triangleq \{x \in \mathbb{F}^{m\times n}: \|x-c\| < \epsilon\}$. Let $\mathbb{Z}^+$ denote the set of positive integers.

Definition 1. Let $\Delta \subseteq \mathbb{F}^{m\times n}$ be bounded and contain no isolated points. For all $j \in \mathbb{Z}^+$, let $\Delta_j \subseteq \Delta$ be a finite set. Then, $\{\Delta_j\}_{j=1}^\infty$ *converges to* Δ if for each $x \in \Delta$, there exists a sequence $\{x_j: x_j \in \Delta_j\}_{j=1}^\infty$ such that for all $\epsilon > 0$, there exists $L \in \mathbb{Z}^+$ such that for all $j > L$, $x_j \in \mathbb{B}_\epsilon(x)$.

3. Problem formulation

Let $G_y: \mathbb{C} \to \mathbb{C}^{n\times m}$ and $G_v: \mathbb{C} \to \mathbb{C}^{l\times m}$ be real rational transfer function matrices, and consider the linear time-invariant system

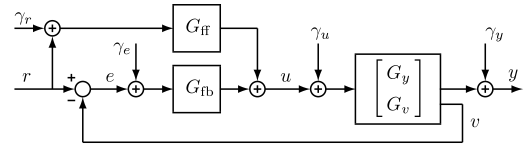

Fig. 2. The input r and output y are measured, but all internal signals and the noises are unmeasured.

$$y(z) = G_y(z)[u(z) + \gamma_u(z)] + \gamma_y(z), \tag{1}$$
$$v(z) = G_v(z)[u(z) + \gamma_u(z)], \tag{2}$$

where $y(z) \in \mathbb{C}^n$, $\gamma_y(z) \in \mathbb{C}^n$, $u(z) \in \mathbb{C}^m$, $\gamma_u(z) \in \mathbb{C}^m$, and $v(z) \in \mathbb{C}^l$ are the z-transforms of the output, output noise, control, control noise, and feedback, respectively. The control u is generated by feedback and feedforward as shown in Fig. 2. Let $G_{\mathrm{ff}}, G_{\mathrm{fb}}: \mathbb{C} \to \mathbb{C}^{m\times l}$ be real rational transfer function matrices, and consider the control

$$u(z) = G_{\mathrm{ff}}(z)[r(z) + \gamma_r(z)] + G_{\mathrm{fb}}(z)[e(z) + \gamma_e(z)], \tag{3}$$

where $r(z) \in \mathbb{C}^l$ is the exogenous input, $\gamma_r(z) \in \mathbb{C}^l$ is the feedforward noise, $e(z) \triangleq r(z) - v(z)$ is the error, and $\gamma_e(z) \in \mathbb{C}^l$ is the error noise. We assume that G_{ff} is asymptotically stable, that is, the poles of G_{ff} are contained in the open unit disk. The closed-loop system obtained from (1)–(3) is

$$y(z) = \tilde{G}(z)r(z) + \gamma(z),$$

where

$$\tilde{G} \triangleq G_y(I_m + G_{\mathrm{fb}}G_v)^{-1}(G_{\mathrm{fb}} + G_{\mathrm{ff}}) \tag{4}$$

is assumed to be asymptotically stable, and the noise is

$$\gamma \triangleq G_y(I_m + G_{\mathrm{fb}}G_v)^{-1}(G_{\mathrm{ff}}\gamma_r + G_{\mathrm{fb}}\gamma_e - G_{\mathrm{fb}}G_v\gamma_u) + G_y\gamma_u + \gamma_y.$$

Let $N \in \mathbb{Z}^+$ be the number of frequency response data, and define $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. For all $k \in \mathcal{N}$, let $\theta_k \in [0, \pi]$, where $\theta_1 < \cdots < \theta_N$. Define the *closed-loop frequency response data*

$$H(\theta_k) \triangleq \tilde{G}(e^{\jmath\theta_k}) + \Gamma(e^{\jmath\theta_k}) \in \mathbb{C}^{n\times l}, \tag{5}$$

where $\Gamma: \mathbb{C} \to \mathbb{C}^{n\times l}$ is such that, for all $i \in \{1, 2, \ldots, n\}$ and all $j \in \{1, 2, \ldots, l\}$, $\Gamma_{(i,j)} \triangleq \gamma_{(i)}/r_{(j)}$. Define the noise matrix

$$\eta_* \triangleq [\Gamma(\sigma_1) \quad \cdots \quad \Gamma(\sigma_N)] \in \mathbb{C}^{n\times lN}.$$

This paper presents an SSID method to identify G_{ff} and G_{fb} under the assumption that G_y, G_v, and $\{H(\theta_k)\}_{k=1}^N$ are known. For each $k \in \mathcal{N}$, $H(\theta_k)$ can be calculated from y and r as $H_{(i,j)}(\theta_k) = y_{(i)}(e^{\jmath\theta_k})/r_{(j)}(e^{\jmath\theta_k})$. Thus, $\{H(\theta_k)\}_{k=1}^N$ can be obtained from the accessible signals r and y, and does not depend on the internal signals (e.g., u and v) or the noise signals γ_r, γ_e, γ_u, and γ_y, which are not assumed to be measured.

We assume that G_{ff} is FIR. Thus, we can express the feedforward transfer function matrix as $G_{\mathrm{ff}}(z) = z^{-n_{\mathrm{ff}}}N_{\mathrm{ff}}(z)$, where $N_{\mathrm{ff}} \in \mathbb{R}^{m\times l}[z]$ and $n_{\mathrm{ff}} \triangleq \deg N_{\mathrm{ff}}$. Since G_{ff} is asymptotically stable, it follows that for sufficiently large order n_{ff}, G_{ff} can approximate an infinite impulse response (IIR) transfer function matrix to arbitrary accuracy evaluated along the unit circle. Thus, the assumption that G_{ff} is FIR does not significantly restrict the class of feedforward behavior. The SSID approach in this paper can also be used with an IIR feedforward model, but using an FIR feedforward model improves computational efficiency as discussed in Section 7.

Let G_y and G_v have the right-matrix-fraction descriptions $G_y = N_yD^{-1}$ and $G_v = N_vD^{-1}$, and let G_{fb} have the left-matrix-fraction description $G_{\mathrm{fb}} = D_{\mathrm{fb}}^{-1}N_{\mathrm{fb}}$, where $N_y \in \mathbb{R}^{n\times m}[z]$, $N_v \in \mathbb{R}^{l\times m}[z]$,