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Brief paper

A constructive procedure for energy shaping of port–Hamiltonian systems[☆]

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ABSTRACT

Equilibrium stabilization of nonlinear systems via energy shaping is a well-established, robust, passivity-based controller design technique. Unfortunately, its application is often stymied by the need to solve partial differential equations, which is usually a difficult task. In this paper a new, *fully constructive*, procedure to shape the energy for a class of *port-Hamiltonian* systems that *obviates* the solution of partial differential equations is proposed. Proceeding from the well-known passive, *power shaping* output we propose a nonlinear static state-feedback that preserves passivity of this output but with a *new storage function*. A suitable selection of a controller gain makes this function positive definite, hence it is a suitable Lyapunov function for the closed-loop. The resulting controller may be interpreted as a classical PI—connections with other standard passivity-based controllers are also identified.

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1. Introduction

An energy shaping controller for *mechanical systems* that does not require the solution of partial differential equations (PDEs) has been recently proposed in Donaire et al. (2016). In this paper we pursue this research line considering the more general case of *port-Hamiltonian* (pH) systems (van der Schaft, 1996).

The starting point of the design is the well-known *power shaping* output (Ortega, Jeltsema, & Scherpen, 2003), which defines a passive output for the pH system with storage function its energy function. As is well-known a PI controller around this output preserves the passivity of the closed-loop. It is then shown that, if the power shaping output is "integrable", the integral action of the PI is passive with a storage function a quadratic term of the "integral" of the power shaping output, which depends on the plant state. In this way we can generate a *new storage function* for the closed-loop constructed as the sum of this function and the original energy function of the pH system. Adding a suitably chosen constant to the control makes this function *positive definite*, which then qualifies as a Lyapunov function for the closed-loop system. The condition imposed on the power shaping output boils down to a classical integrability condition of some computable vector fields, hence it can be readily verified.

The remainder of the paper is organized as follows. Section 2 presents the problem formulation and main assumptions on the pH system. The design of the energy shaping feedback is carried out in Section 3. Section 4 contains the main stabilization result. Section 5 explores the relationship between the new controller and the well-known PBC techniques of energy balancing (EB-PBC) (Ortega, van der Schaft, Castanos, & Astolfi, 2008; Ortega, van der Schaft, Mareels, & Maschke, 2001) and interconnection and damping assignment (IDA-PBC) (Ortega & García-Canseco, 2004). In Section 6 we show that the method is applicable to a benchmark example and consider the case of LTI systems. The paper is wrapped-up with concluding remarks in Section 7. A preliminary version of this paper has been reported in Borja, Cisneros, and Ortega (2015).

Notation. For $x \in \mathbb{R}^n$, $S \in \mathbb{R}^{n\times n}$, $S = S^\top > 0$, we denote $|x|^2 := x^\top x$, and $\|x\|_S^2 := x^\top S x$. For a full-rank matrix $g \in \mathbb{R}^{n\times m}$ we define $g^\dagger := (g^\top g)^{-1} g^\top$. All the functions in the paper are assumed smooth. For $H : \mathbb{R}^n \to \mathbb{R}$ we denote $\nabla H(x) = \left(\frac{\partial H(x)}{\partial x}\right)^\top$. For $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$, we define $\nabla \mathcal{C}(x) = [\nabla \mathcal{C}_1(x), \ldots, \nabla \mathcal{C}_m(x)]$.

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For any mapping $F : \mathbb{R}^n \to \mathbb{R}^{p\times q}$ and the distinguished element $x_* \in \mathbb{R}^n$ we define the constant matrix $F_* := F(x_*)$.

2. Problem formulation and main assumptions

The standard input-state representation of pH systems is of the form van der Schaft (1996)

$$\dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)]\nabla H(x) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$, $m \leq n$, is the control vector, $H : \mathbb{R}^n \to \mathbb{R}$ is the systems Hamiltonian, $\mathcal{J}, \mathcal{R} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, with $\mathcal{J}(x) = -\mathcal{J}^\top(x)$ and $\mathcal{R}(x) = \mathcal{R}^\top(x) \geq 0$, are the interconnection and damping matrices, respectively, and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the input matrix, which is full rank. To simplify the notation in the sequel we define the matrix $F : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $F(x) := \mathcal{J}(x) - \mathcal{R}(x)$.

The *control objective* is to stabilize an equilibrium x_*, which is an element of the set of assignable equilibria defined as

$$\mathcal{E} := \left\{x \in \mathbb{R}^n \mid g^\perp(x)F(x)\nabla H(x) = 0\right\}, \tag{2}$$

where $g^\perp : \mathbb{R}^n \to \mathbb{R}^{(n-m)\times n}$ is a full-rank, left-annihilator of $g(x)$, that is, $g^\perp(x)g(x) = 0$ and rank $\{g^\perp(x)\} = n - m$.

The following assumptions identify the class of pH systems for which the proposed control strategy is applicable.

Assumption 1. The matrix $F(x)$ is *full rank*.

Assumption 2. The vector fields $F^{-1}(x)g_i(x)$, with $g_i(x)$, $i = 1, \ldots, m$, the columns of the matrix $g(x)$, are *gradient vector fields*. That is,

$$\nabla\left(F^{-1}(x)g_i(x)\right) = \left[\nabla\left(F^{-1}(x)g_i(x)\right)\right]^\top.$$

If Assumption 1 holds, it is possible to define the *power shaping output* as follows

$$y_{\rm PS} := -g^\top(x)F^{-\top}(x)\left[F(x)\nabla H(x) + g(x)u\right]. \tag{3}$$

As shown in Ortega et al. (2003) and Ortega et al. (2008) $y_{\rm PS}$ is a cyclo-passive output[1] for the pH system (1) with storage function $H(x)$. More precisely, the following dissipation inequality holds

$$\dot{H} \leq u^\top y_{\rm PS}. \tag{4}$$

Noting that $y_{\rm PS}$ may be written as

$$y_{\rm PS} = -g^\top(x)F^{-\top}(x)\dot{x}. \tag{5}$$

Hence, recalling Poincare's lemma it is easy to see that Assumption 2 ensures the existence of a mapping $\gamma : \mathbb{R}^n \to \mathbb{R}^m$ such that

$$\dot{\gamma} = (\nabla\gamma)^\top\dot{x} = y_{\rm PS}. \tag{6}$$

3. Energy shaping

In this section we define a static state-feedback such that the system (1) in closed-loop with this control preserves passivity of the mapping $v \mapsto y_{\rm PS}$ but with a suitably *modified storage function*.

Proposition 1. *Suppose Assumptions 1 and 2 hold. Define the mapping* $u_{\rm PS} : \mathbb{R}^n \to \mathbb{R}^m$

$$u_{\rm PS}(x) := [I - K_P g^\top(x)F^{-\top}(x)g(x)]^{-1}\left[-K_I(\gamma(x) + C) + K_P g^\top(x)F^{-\top}(x)F(x)\nabla H(x)\right] \tag{7}$$

where[2]

$$\nabla\gamma(x) := -F^{-1}(x)g(x) \tag{8}$$

and $C \in \mathbb{R}^m$ *and* $K_P, K_I \in \mathbb{R}^{m\times m}$, $K_I, K_P > 0$, *are free parameters. The system (1) in closed-loop with the control* $u = u_{\rm PS}(x) + v$ *defines a cyclo-passive mapping* $v \mapsto y_{\rm PS}$ *with storage function*

$$H_d(x) = H(x) + \frac{1}{2}\|\gamma(x) + C\|^2_{K_I}. \tag{9}$$

Proof. To establish the proof, first, notice that from (5) and (6) the control (7) reduces to

$$u_{\rm PS}(x) = -\left[K_I(\gamma + C) + K_P y_{\rm PS}\right]. \tag{10}$$

Therefore, differentiating (9) we get

$$\begin{aligned}\dot{H}_d &= \dot{H} + y_{\rm PS}^\top K_I(\gamma + C)\\ &\leq y_{\rm PS}^\top\left[u + K_I(\gamma + C)\right]\\ &= y_{\rm PS}^\top(v - K_P y_{\rm PS}) \leq y_{\rm PS}^\top v,\end{aligned}$$

where we used (6) in the first equality, (4) in the first inequality, (10) for the second equality, respectively, and $K_P > 0$ for the last inequality. □

Remark 1. From (10) and (6) it is easy to see that the control $u = u_{\rm PS}(x) + v$ is a PI of the form

$$u(t) = -K_P y_{\rm PS}(t) - K_I\int_0^t y_{\rm PS}(\tau)d\tau + D + v, \tag{11}$$

where $D := -K_I[C + \gamma(x(0))]$. This establishes the connection with the PI-like controller proposed for mechanical systems in Donaire et al. (2016). Notice that in its state space realization (7) the pH structure is not preserved in closed-loop. However, it does if the PI (11) is implemented via a dynamic extension.

Remark 2. The condition of integrability of the vector fields $F^{-1}(x)g_i(x)$ appears also in the context of Control-by-Interconnection of pH systems, as a necessary and sufficient condition for existence of Casimir functions, see Ortega and Borja (2014) for further details.

4. Stabilization

From Proposition 1 it is clear that if the new storage function $H_d(x)$ is positive definite (with respect to the desired equilibrium x_*) it qualifies as a *bona fide* Lyapunov function for the closed-loop system (with $v = 0$) that ensures stability of x_*. This fact is stated in the proposition below where we also give an easily verifiable condition to check positivity of $H_d(x)$.

Proposition 2. *Consider the system (1), verifying Assumptions 1 and 2, in closed-loop with the control* $u = u_{\rm PS}(x)$, *where* $u_{\rm PS}(x)$ *is given by (7). Fix*

$$C := K_I^{-1}g_*^\dagger F_*(\nabla H)_* - \gamma_*. \tag{12}$$

[1] We recall that, in contrast to passive systems where the storage function is bounded from below, in cyclo-passive systems this is not necessary (Duindam, Macchelli, Stramigioli, & Bruyninckx, 2009).

[2] Notice that the existence of $\gamma(x)$ is ensured by Assumption 2 and it can be computed via direct integration.

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