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Enhancement of adaptive observer robustness applying sliding mode techniques[☆]

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ABSTRACT

The problem studied in this paper is one of improving the performance of a class of adaptive observer in the presence of exogenous disturbances. The H_∞ gains of both a conventional and the newly proposed sliding-mode adaptive observer are evaluated, to assess the effect of disturbances on the estimation errors. It is shown that if the disturbance is "matched" in the plant equations, then including an additional sliding-mode feedback injection term, dependent on the plant output, improves the accuracy of observation.

1. Introduction

The problem of adaptive observer design for nonlinear systems is an active research topic that finds many applications in various engineering fields. Typically, the observer needs to generate estimates of the vector of unknown parameters and unmeasured state components under noisy environments (see for example Besançon, 2007, Ioannou & Sun, 1996). High-gain observers with gain adaptation for time-varying or nonlinear systems have been studied in a number of recently published papers, see for instance Alessandri and Rossi (2013), Boizot, Busvelle, and Gauthier (2010), Farza, Bouraoui, Ménard, Ben Abdennour, and M'Saad (2014) and Sanfelice and Praly (2011).

An important issue is the *relative degree* between the output signal and the vector of unknown parameters (*i.e.* the number of derivatives of the output required, before the direct dependence on the vector of unknown parameters is obtained). Observers designed in the case when the degree is one (Fradkov, Nijmeijer, & Markov, 2000) and for the higher relative degree case (Efimov, 2004; Fradkov, Nikiforov, & Andrievsky, 2002; Xu & Zhang, 2004; Zhang, 2002) have completely different structures, and the dimension of the observers in the latter case is much higher.

There exist a number of potential solutions aimed at improving the robustness in nonlinear systems by applying dynamic or static feedback. Some very promising solutions have been obtained in the area of sliding mode theory, since sliding mode feedback is able to fully compensate for matched disturbances granting the closed loop system finite-time stability (Shtessel, Edwards, & Fridman, 2013). Recently the sliding mode approach has been successfully applied to adaptive observer design in the case of relative degree one systems (Yan & Edwards, 2008), but the extension of this theory for adaptive observer design with a high relative degree is complicated due to the fixed observer structure.

In this paper a method is presented for augmenting the adaptive observer from Zhang (2002), using sliding mode feedback, to cope with matched uncertainties in the spirit of Yan and Edwards (2008). The resulting solution ensures that the level of observer robustness with respect to some matched disturbances is improved.

2. Problem statement

Consider the following uncertain nonlinear system:

$$\dot{x} = Ax + \phi(y, u) + G(y, u)\theta + Bv, \quad y = Cx, \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$ are the state, the output and the control respectively, $\theta \in \mathbb{R}^q$ is the vector of unknown parameters;

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$\upsilon \in \mathbb{R}^s$ is the vector of external disturbances and $\upsilon : \mathbb{R}_+ \to \mathbb{R}^s$ is a (Lebesgue) measurable function of time; the matrices A, B, C are known and are assumed to have appropriate dimensions (and the pair (A, C) is detectable); the functions $\phi : \mathbb{R}^{p+m} \to \mathbb{R}^n$ and $G : \mathbb{R}^{p+m} \to \mathbb{R}^{n\times q}$ are also assumed to be known and ensure uniqueness and existence of solutions to system (1) at least locally.

The symbol $|x|$ denotes the Euclidean norm of a vector x (for a matrix A the symbol $|A|$ denotes the induced matrix norm), and for the (Lebesgue) measurable functions $\upsilon : \mathbb{R}_+ \to \mathbb{R}^s$, the norm is defined as $\|\upsilon\| = \text{ess sup}_{t\geq 0}\{|\upsilon(t)|\}$. For a matrix function $A : \mathbb{R}_+ \to \mathbb{R}^{s\times q}$ we denote $\|A\| = \| |A(t)| \|$. The identity matrix of dimension $n \times n$ is denoted as I_n and the symbols $\lambda_{\min}(A)$, $\lambda_{\max}(A)$ represent the minimal and maximal eigenvalues of a symmetric matrix $A \in \mathbb{R}^{n\times n}$.

In this work we will assume that certain signals in the system (1) are bounded:

Assumption 1. $\|\upsilon\| < +\infty$, $\|G(y, u)\| < +\infty$.

Although assumed bounded, the disturbance υ may have a large magnitude, and therefore special attenuation techniques have to be applied to ensure reliable estimates for the states in system (1).

The objective of this work is to design an adaptive observer for (1) under the conditions of Assumption 1. The observer has to provide estimates of the vectors x and θ with an enhanced degree of robustness with respect to the external disturbance υ. The proposed design procedure is completed in two steps. Firstly, an adaptive observer is designed and the H_∞ gain between the disturbances and the output errors is calculated. Secondly, an additional sliding mode output injection is applied to further reduce the influence of the disturbance components which cannot be completely rejected by the first step.

An adaptive observer for the system (1) has been proposed in Zhang (2002), and takes the form:

$$\dot{z} = Az + \phi(y, u) + G(y, u)\hat{\theta} + L(y - Cz) + \Omega\dot{\hat{\theta}}, \tag{2}$$

where

$$\dot{\Omega} = (A - LC)\Omega + G(y, u), \qquad \dot{\hat{\theta}} = \gamma\Omega^T C^T(y - Cz). \tag{3}$$

In (2)–(3), $z \in \mathbb{R}^n$ is the estimate of x, $\hat{\theta} \in \mathbb{R}^q$ is the estimate of θ, and $\Omega \in \mathbb{R}^{n\times q}$ is an auxiliary/filter variable, that helps to overcome possible high relative degree obstructions in system (1). In (3) $\gamma > 0$ is a design parameter, and L is the observer gain that is chosen to ensure a Hurwitz property for the matrix $A - LC$. The analysis of the estimation abilities of the observer in (2), (3) is based on the errors $\delta = x - z + \Omega\tilde{\theta}$ and $\tilde{\theta} = \hat{\theta} - \theta$ whose dynamics can be shown to have the form:

$$\dot{\delta} = (A - LC)\delta + B\upsilon, \tag{4}$$

$$\dot{\tilde{\theta}} = \gamma\Omega^T C^T(C\delta - C\Omega\tilde{\theta}). \tag{5}$$

From Eq. (4) we conclude that $\delta(t) \underset{t\to+\infty}{\longrightarrow} 0$ for $\upsilon = 0$ and the variable δ stays bounded for any bounded disturbance υ. From (3) the Hurwitz property of the matrix $A - LC$ and Assumption 1 imply boundedness of the variable Ω. If the signal $G(y, u)$ is persistently exciting (PE) (Anderson, 1977; Yuan & Wonham, 1977), then due to the filtering property of the variable Ω, the variable $C\Omega$ is also PE. Moreover, it is possible to show (Efimov, 2004) that for any bounded signal $C\delta$, the variable $\tilde{\theta}$ has a bounded response, and if $C\delta(t) \underset{t\to+\infty}{\longrightarrow} 0$, then $\tilde{\theta}(t) \underset{t\to+\infty}{\longrightarrow} 0$ also. This "proof" is based on general stability arguments, and no strict Lyapunov function has been proposed (this drawback will be overcame later in the present work).

3. Conventional adaptive observer

First let us show that the system in (5) is input-to-state stable with respect to the input $C\delta$.

Lemma 1 (*Efimov & Fradkov, 2015*)**.** *Let the variable $\Omega^T C^T$ be PE and bounded,* i.e. $0 < \rho = \|C\Omega\| < +\infty$ *and there exist constants $\vartheta > 0$ and $\ell > 0$ such that*

$$\int_t^{t+\ell} \Omega(\tau)^T C^T C\Omega(\tau)d\tau \geq \vartheta I_q \quad \forall t \geq 0.$$

Then

(a) *there exists a continuous symmetric matrix function $P : \mathbb{R}_+ \to \mathbb{R}^{q\times q}$ such that $\rho^{-2}I_q \leq 2\gamma P(t) \leq \alpha I_q$ for all $t \geq 0$, where the scalar $\alpha = \gamma\eta^{-1}e^{2\eta\ell}$ and $\eta = -0.5\ell^{-1}\ln(1 - \frac{\gamma\vartheta}{1+\gamma^2\ell^2\rho^4})$;*

(b) *for all $t \geq 0$*

$$\dot{P}(t) - \gamma P(t)\Omega(t)^T C^T C\Omega(t) - \gamma\Omega(t)^T C^T C\Omega(t)P(t) + I_q = 0;$$

(c) *for $S(t, \tilde{\theta}) = \tilde{\theta}^T P(t)\tilde{\theta}$ we have for all $\tilde{\theta} \in \mathbb{R}^q$, $\delta \in \mathbb{R}^n$ and $t \geq 0$*

$$\dot{S} \leq -\gamma\alpha^{-1}S + 0.5\rho^2\alpha^2|C\delta|^2.$$

In addition, for all $\tilde{\theta}(0) \in \mathbb{R}^q$ and all $t \geq 0$ the following estimate is satisfied:

$$|\tilde{\theta}(t)| \leq \rho\sqrt{\alpha}[e^{-0.5\gamma\alpha^{-1}t}|\tilde{\theta}(0)| + \rho\alpha\|C\delta\|].$$

Remark 2. This lemma also provides an estimate on the fastest rate of decrease of the parametric estimation error $\tilde{\theta}(t)$. Specifically, the rate of decrease equals $0.5\gamma\alpha^{-1} = 0.5\eta e^{-2\eta\ell} = -0.25\ell^{-1}\ln(1 - \frac{\gamma\vartheta}{1+\gamma^2\ell^2\rho^4})(1 - \frac{\gamma\vartheta}{1+\gamma^2\ell^2\rho^4}) = g(\gamma)$. The mapping g is a function of γ dependent on parameters $\vartheta > 0$, $\ell > 0$ and $0 < \rho = \|C\Omega\| < +\infty$ of the PE variable $\Omega^T C^T$. Computing the derivative of g we obtain:

$$\frac{\partial g}{\partial\gamma} = 0.25\ell^{-1}\vartheta\frac{1 - \gamma^2\ell^2\rho^4}{(1 + \gamma^2\ell^2\rho^4)^2}\left(1 + \ln\left(1 - \frac{\gamma\vartheta}{1 + \gamma^2\ell^2\rho^4}\right)\right).$$

Since $\vartheta \leq \ell\rho^2$ from the definition of the PE property, then the equation $\frac{\partial g}{\partial\gamma} = 0$ has just one solution

$$\gamma_{\text{opt}} = \ell^{-1}\rho^{-2},$$

which gives the maximum rate of convergence $g(\gamma_{\text{opt}}) = -\frac{1}{4\ell}\ln(1 - \frac{\vartheta}{2\ell\rho^2})(1 - \frac{\vartheta}{2\ell\rho^2})$ of the parametric estimation error $\tilde{\theta}(t)$. Increasing the convergence rate implies a decrease in α, and also decreases the value of the H_∞ gain.

Note that in order to use these estimates we have to know the values of ρ, ℓ and ϑ. The *existence* of such a ρ follows from Assumption 1 for the Hurwitz matrix $A - LC$, but to compute it we have to know $\|x\|$, which is assumed to be unavailable. However, the values of ρ, ℓ and ϑ can all be evaluated numerically during experiments by computing the integral of $\Omega(t)^T C^T C\Omega(t)$ (the values ℓ and ϑ may be not unique, but fixing one of them imposes a value on the another).

Theorem 3. *Suppose Assumption 1 is satisfied, the variable $\Omega^T C^T$ is PE,* i.e. *there exist constants $\vartheta > 0$ and $\ell > 0$ such that*

$$\int_t^{t+\ell} \Omega(\tau)^T C^T C\Omega(\tau)d\tau \geq \vartheta I_q \quad \forall t \geq 0,$$

and there exists a $n \times n$ matrix $W = W^T > 0$ and constants $r > 0$, $h > 0$ such that

$$(A - LC)^T W + W(A - LC) + 0.5r\alpha^2 C^T C + hWBB^T W + \gamma\alpha^{-1}W \leq 0, \tag{6}$$