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Fault tolerant quantum filtering and fault detection for quantum systems ☆

Qing Gao, Daoyi Dong [1], Ian R. Petersen

School of Engineering and Information Technology, University of New South Wales, Canberra, ACT, 2600, Australia

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ABSTRACT

This paper aims to determine the fault tolerant quantum filter and fault detection equation for a class of open quantum systems coupled to a laser field that is subject to stochastic faults. In order to analyze this class of open quantum systems, we propose a quantum–classical Bayesian inference method based on the definition of a so-called quantum–classical conditional expectation. It is shown that the proposed Bayesian inference approach provides a convenient tool to simultaneously derive the fault tolerant quantum filter and the fault detection equation for this class of open quantum systems. An example of two-level open quantum systems subject to Poisson-type faults is presented to illustrate the proposed method. These results have the potential to lead to a new fault tolerant control theory for quantum systems.

1. Introduction

The theory of filtering, which in a broad sense is a scheme considering the estimation of the system states from noisy signals and/or partial observations, plays a significant role in modern engineering science. A filter propagates our knowledge about the system states given all observations up to the current time and provides optimal estimates of the system states. From the fundamental postulates of quantum mechanics, one is not allowed to make noncommutative observations of quantum systems in a single realization or experiment. Any quantum measurement yields in principle only partial information about the system. This fact makes the theory of quantum filtering extremely useful in measurement based feedback control of quantum systems, especially in the field of quantum optics (Rouchon & Ralph, 2015; Wiseman & Milburn, 2010). A system–probe interaction setup in quantum optics is used as the typical physical scenario concerning the extraction of information about the quantum system from continuous measurements (Belavkin, 1992; Gardiner & Zoller, 2000). The quantum system under consideration, e.g., a cloud of atoms trapped inside a vacuum chamber, is interrogated by probing it with a laser beam. After interaction with the electromagnetic radiation (laser), the free electrons of the atoms are accelerated and can absorb energy. This energy is then emitted into the electromagnetic field as photons which can be continuously detected through a homodyne detector (Wiseman & Milburn, 2010). Using the continuous integrated photocurrent generated by the homodyne detector one can conveniently estimate the atomic observables. To find the optimal estimates is then precisely the goal of quantum filtering theory. A very early approach to quantum filtering was presented in a series of papers by Belavkin dating back to the early 1980s (Belavkin, 1980, 1992), which was developed in the framework of continuous nondemolition quantum measurement using the operational formalism from Davies's precursor work (Davies, 1969). In the physics community, the theory of quantum filtering was also independently developed in the early 1990s (Carmichael, 1993), named "quantum trajectory theory" in the context of quantum optics.

Particular emphasis is given to the work by Bouten, van Handel, and James (2007) where quantum probability theory was used in a rigorous way and a quantum filter for a laser–atom interaction setup in quantum optics was derived using a quantum reference probability method. A basic idea in quantum probability theory is an isomorphic equivalence between a commutative subalgebra of quantum operators on a Hilbert space and a

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E-mail addresses: qing.gao.chance@gmail.com (Q. Gao), daoyidong@gmail.com (D. Dong), i.r.petersen@gmail.com (I.R. Petersen).

[1] Tel.: +61 2 62686285; fax: +61 2 62688443.

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classical (Kolmogorov) probability space through the spectral theorem, from which any probabilistic quantum operation within the commutative subalgebra can be associated with its classical counterpart. The complete quantum probability model is treated as the noncommutative counterpart of Kolmogorov's axiomatic characterization of classical probability. Similar to the classical case (Bertsekas & Tsitsiklis, 2002), the optimal estimate of any observable is given by its quantum expectation conditioned on the history of continuous nondemolition quantum measurements of the electromagnetic field. The quantum filter was derived in terms of *Itô* stochastic differential equations using a reference probability method.

In practice, classical randomness may be introduced directly into the system dynamics of quantum systems (Ruschhaupt, Chen, Alonso, & Muga, 2012). For example, the system Hamiltonian of a superconducting quantum system may contain classical randomness due to the existence of stochastic fluctuations in magnetic flux or gate voltages (Dong, Chen, Qi, Petersen, & Nori, 2015). A spin system may be subject to stochastically fluctuating fields that will introduce classical randomness into the system dynamics (Dong & Petersen, 2012). For an atom system subject to a laser beam, the occurrence of stochastic faults in the laser device may cause the introduction of classical randomness into the dynamics of the atom system (Khodjasteh & Lidar, 2005; Viola & Knill, 2003). For an open quantum system, the system may evolve randomly and the system dynamics may involve two kinds of randomnesses, i.e., *quantum randomness* due to intrinsic quantum indeterminacy and *classical randomness* arising from the imprecise behavior of macroscopic devices. In order to solve this issue, Bouten, van Handel, and James (2009) presented an approach to analyzing quantum observables containing classical random information. By using quantum spectral theorem, a classical random variable was equivalently represented by a quantum observable in a commutative quantum probability space on an external Hilbert space. As a result, a random observable can be interpreted by compositing an operator-valued function with this quantum observable and can be well defined on an enlarging quantum probability space. In order to estimate classical random parameters from quantum measurements, joint quantum and classical statistics were also considered in literature using the concept of "hybrid" classical–quantum density operator, see e.g., (Dotsenko et al., 2009; Gambetta & Wiseman, 2001; Kato & Yamamoto, 2013; Negretti & Mølmer, 2013; Somaraju, Dotsenko, Sayrin, & Rouchon, 2012; Tsang, 2009a,b, 2010). In this paper, we concentrate on a class of open quantum systems subject to stochastic faults, aiming at deriving the fault tolerant quantum filtering equation and the fault detection equation (Gao, Dong, & Petersen, 2015). In order to achieve this goal, we consider an approach to uniformly analyzing quantum observables and classical random variables. First, the isomorphic equivalent relationship between a set of random observables equipped with a quantum–classical expectation operation and a classical probability space model is determined. Then a quantum–classical conditional expectation is considered using the associated classical concept, based on which a Bayes formula is obtained. This Bayesian inference method provides a convenient tool to simultaneously derive the fault tolerant quantum filter and fault detection equations for this class of systems. The obtained equations are given by classical *Itô* differential equations and can be conveniently used in practical implementation.

This paper is organized as follows. Section 2 describes the class of open quantum systems under consideration in this paper. Section 3 is devoted to statistical interpretation of quantum observables containing information of classical random parameters. In Section 4, the fault tolerant quantum filter and fault detection equations are simultaneously derived for open quantum systems using a Bayesian inference method. An example of two-level quantum systems with Poisson-type faults is illustrated. Section 5 concludes this paper.

2. Heisenberg dynamics of open quantum systems

In this work, we concentrate on an open quantum system that has been widely investigated in quantum optics (Qi, Pan, & Guo, 2013; van Handel, Stockton, & Mabuchi, 2005; Wiseman & Milburn, 2010). The quantum system under consideration is a cloud of atoms in weak interaction with an external laser probe field which is continuously monitored by a homodyne detector (Bouten et al., 2007; Mirrahimi & van Handel, 2007). Such a quantum system can be described by quantum stochastic differential equations driven by quantum noises $B(t)$ and $B^\dagger(t)$ (Wiseman & Milburn, 2010). The dynamics of the quantum system are described by the following quantum stochastic differential equation[2]:

$$dU(t) = \left\{\left(-iH(t) - \frac{1}{2}L^\dagger L\right)dt + LdB^\dagger(t) - L^\dagger dB(t)\right\}U(t), \qquad (1)$$

with initial condition $U(0) = I$ and $i = \sqrt{-1}$. Here $U(t)$ describes the Heisenberg-picture evolution of the system operators and $H(t)$ is the system Hamiltonian. In terms of the system states, if π_0 is a given system state, we write $\rho_0 = \pi_0 \otimes |\upsilon\rangle\langle\upsilon|$, where $|\upsilon\rangle$ represents the vacuum state. The system operator L, together with the field operator $b(t) = \dot{B}(t)$ models the interaction between the system and the field. From quantum *Itô* rule, one has (Gardiner & Zoller, 2000)

$$dB(t)dB^\dagger(t) = dt,$$
$$dB^\dagger(t)dB(t) = dB(t)dB(t) = dB^\dagger(t)dB^\dagger(t) = 0.$$

The atom system and the laser field form a composite system and the Hilbert space for the composite system is given by $\mathcal{H}_\mathcal{S} \otimes \mathcal{E} = \mathcal{H}_\mathcal{S} \otimes \mathcal{E}_{t]} \otimes \mathcal{E}_{(t}$ where we have exhibited the continuous temporal tensor product decomposition of the Fock space $\mathcal{E} = \mathcal{E}_{t]} \otimes \mathcal{E}_{(t}$ into the past and future components (Belavkin, 1992; Holevo, 1991). It is assumed that $\dim(\mathcal{H}_\mathcal{S}) = n < \infty$. The atomic observables are described by self-adjoint operators on $\mathcal{H}_\mathcal{S}$. Any system observable X at time t is given by $X(t) = j_t(X) = U^\dagger(t)(X \otimes I)U(t)$. It is noted that (1) is written in *Itô* form, as will all stochastic differential equations in this paper.

In practice, the system Hamiltonian may change randomly because of, e.g., faulty control Hamiltonians that appear in the system dynamics at random times (Khodjasteh & Lidar, 2005; Viola & Knill, 2003) or random fluctuations of the external electromagnetic field (Dong et al., 2015; Ruschhaupt et al., 2012). In this case, the system Hamiltonian can be described by a Hermitian operator $H(F(t))$ that depends on some classical stochastic process $F(t)$. Using the quantum *Itô* rule (Hudson & Parthasarathy, 1984), one has $d(U^\dagger(t)U(t)) = d(U(t)U^\dagger(t)) = 0$, which implies that $U(t)$ is a *random unitary operator* and $X(t) = j_t(X)$ is a *random observable*, both depending on the stochastic process $F(t)$. In this paper, for simplicity we still write $U(t)$ instead of the functional form $U(F, t)$. One can conclude that the commutativity of observables is preserved, that is, $[j_t(A), j_t(B)] = 0$ if $[A, B] = 0$ where A, B are two system observables in $\mathcal{H}_\mathcal{S}$. Here the commutator is defined by $[A, B] = AB - BA$. In addition, from (1) one can see that $U(t)$ depends on $B(t')$ and $B^\dagger(t')$, $0 \le t' < t$, since the increments $dB(t)$ and $dB^\dagger(t)$ point to the future evolution. Consequently,

$$[U(t), dB(t)] = [U(t), dB^\dagger(t)] = 0. \qquad (2)$$

Similarly, the time evolution operator $U(t, s) = U(t)U^\dagger(s)$ from time s to time t depends only on the field operators $dB(s')$ and $dB^\dagger(s')$ with $s \le s' \le t$. Thus,

$$[U(t, s), B(\tau)] = [U(t, s), B^\dagger(\tau)] = 0, \quad \tau \le s. \qquad (3)$$

[2] We have assumed $\hbar = 1$ by using atomic units in this paper.