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Continuous-time identification of periodically parameter-varying state space models ☆

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ABSTRACT

This paper presents a new frequency domain identification technique to estimate multivariate Linear Parameter-Varying (LPV) continuous-time state space models, where a periodic variation of the parameters is assumed or imposed. The main goal is to obtain an LPV state space model suitable for control, from a single parameter-varying experiment. Although most LPV controller synthesis tools require continuous time state space models, the identification of such models is new. The proposed identification method designs a periodic input signal, taking the periodicity of the parameter variation into account. We show that when an integer number of periods is observed for both the input and the scheduling, the state space model representation has a specific, sparse structure in the frequency domain, which is exploited to speed up the estimation procedure. A weighted non-linear least squares algorithm then minimizes the output error. Two initialization methods are explored to generate starting values. The first approach uses a Linear Time-Invariant (LTI) approximation. The second estimates a Linear Time-Variant (LTV) input–output differential equation, from which a corresponding state space realization is computed.

1. Introduction

Although the Linear Time-Invariant framework (Pintelon & Schoukens, 2012) has given rise to powerful forms of control, the need to operate processes at even higher levels of precision requires more advanced model structures, like non-linear block structured models (Bai & Giri, 2010), time-varying differential equations (Lataire & Pintelon, 2011; Louarroudi, Lataire, Pintelon, Janssens, & Swevers, 2014), and Linear Parameter-Varying models (Rugh & Shamma, 2000; Tóth, 2010). Indeed, most physical systems behave non-linearly, or have a varying dynamic behavior that changes with an external parameter, like the temperature or pressure. Such systems are usually linearized at a chosen operating point. However, in practice it is quite common to utilize the same plant at several set points, each with their own linearized dynamics. A local LPV approach estimates LTI models at a set of operating points, after which a macro-model interpolates the local approximations (Bruzelius & Breitholtz, 2001; De Caigny, Camino, & Swevers, 2011; Ferranti, Knockaert, & Dhaene, 2011). These methods do not incorporate knowledge about the rate of variation of the scheduling parameter and, therefore, the resulting models are only valid in case of slow parameter variations. Contrary to the local approach, we opt for a global identification experiment (Rugh & Shamma, 2000), where the system dynamics are persistently changed by external signals $p(t)$, called the scheduling parameters. The goal is to identify the system from a single parameter-varying input–output experiment.

Modeling of arbitrary time-varying systems is challenging. In the identification phase, we will therefore focus on periodically parameter-varying systems. For example, the steady state operation of a rotating mechanical bearing (Allen, 2009) falls into this class. In other cases, where one has full control over the experimental setup, including the scheduling parameter $p(t)$, the periodicity can be imposed. In process applications, only a perturbation of the associated variables is allowed, due to limitations of actuation and process loss. In many cases, white noise, binary noise, PRBS or step inputs are used to perturb the system. It is also possible

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to use a random phase multisine excitation, resulting in a periodic experiment.

$$\sum_{k=\underline{K}}^{\overline{K}} A_k \cos(2\pi k f_0 t + \psi_k). \tag{1}$$

Here, A_k are user-defined, and ψ_k is uniformly distributed between $[0, 2\pi]$. Alternatively, the amplitudes and phases can be chosen, so that the system trajectory domain is explored optimally. In practice, a random phase multisine signal (1) cannot be discerned from a periodic white noise sequence in the time domain.

1.1. Target application

Most LPV controllers are designed in continuous time, using state space models (Apkarian & Gahinet, 1995; Scherer, 1996; Wu & Dong, 2006), which are given by

$$\dot{x}(t) = A\big(p(t)\big)x(t) + B\big(p(t)\big)u(t) \tag{2}$$

$$y_0(t) = C\big(p(t)\big)x(t) + D\big(p(t)\big)u(t)\,. \tag{3}$$

We define N_x as the size of the state vector $x(t)$ and denote the number of inputs $u(t)$, outputs $y(t)$ and scheduling signals $p(t)$ by N_u, N_y and N_p, respectively. For control design, ideally the coefficients depend only on the instantaneous value of the scheduling, i.e. linear combinations of known/chosen static basis functions in $p(t)$

$$A\big(p(t)\big) = \sum_{i=1}^{N_p} A_i\, \phi_i\big(p(t)\big) \tag{4}$$

where the matrices A_i are constant. A common choice for ϕ_i are the (multivariate) polynomials $p(t)^i$. Similar definitions hold for $B\big(p(t)\big)$, $C\big(p(t)\big)$ and $D\big(p(t)\big)$. The educated guess in the choice of basis function ϕ_i usually follows from the physics of the problem. In practice, the true coefficients will have to be approximated with the proposed basis in $\phi_i\big(p(t)\big)$. In Laurain, Tóth, Zheng, and Gilson (2012), Least Squares Support Vector Machines (LSSVM) are used, while in De Caigny et al. (2011) a polynomial basis is selected. From hereon, we call the collective set of (unknown) observed functions of the scheduling signals $p(t)$.

1.2. Existing work

Since we are essentially solving a non-linear optimization problem, the initial values for the parameters have a big impact on the results. In a first draft of the proposed identification approach (Goos, Lataire, & Pintelon, 2014), an LTI approximation was used, as illustrated in Fig. 1. The results were satisfactory, but when the parameter variation becomes larger or faster, the risk to end up in a local minimum increases. A simple time-invariant model can only approximate a slowly time-varying system. Therefore, in this paper we also examine another initialization routine, which is based on time-varying differential equations. Nowadays, a lot of research is dedicated to the identification of time- (Lataire & Pintelon, 2011; Louarroudi et al., 2014) and parameter-varying (Laurain, Tóth, Gilson, & Garnier, 2010; Tóth, Laurain, Gilson, & Garnier, 2012) differential equations.

$$\sum_{i=1}^{N_a} a_i(t)\, y^{(i)}(t) = \sum_{i=1}^{N_b} b_i(t)\, u^{(i)}(t) \tag{5}$$

$$\sum_{i=1}^{N_a} a_i(p(t)) y^{(i)}(t) = \sum_{i=1}^{N_b} b_i(p(t)) u^{(i)}(t)\,. \tag{6}$$

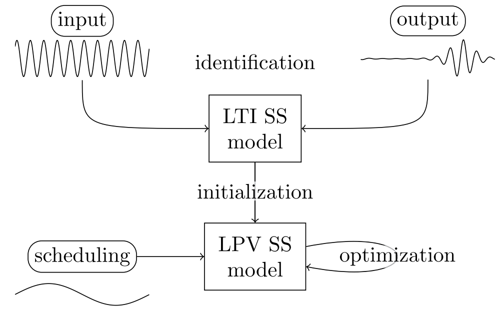

Fig. 1. Originally, the LPV identification was initialized with an LTI approximation. All coefficients related to the parameter variations are set to zero. The optimization searches for a good model, by minimizing the prediction error.

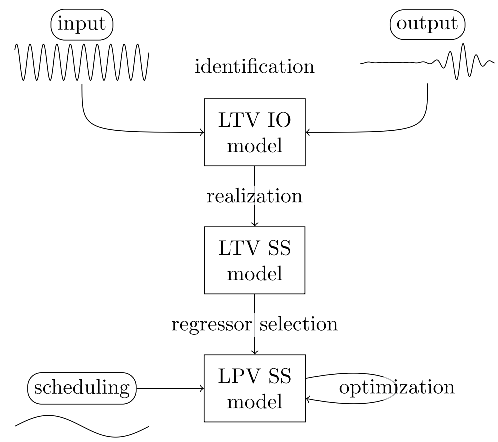

Fig. 2. First a general time-varying input–output model is identified. The corresponding minimal state space model is realized, with a dynamic dependence on the coefficients. Next, we try to establish a link between the known parameter variation $p(t)$ and the time-varying coefficients. Finally, the prediction error is minimized.

From Tóth (2010), we know that it is possible to transform a static parameter-varying differential equation into a minimal state space form, but the resulting models will have a dynamic dependence on the scheduling. From a control perspective, we want a simple, static model, in which the model only depends on the current value of $p(t)$. Therefore, direct application of Tóth (2010) does not yield the desired result.

Recently, we have derived exact computational formulas in the SISO case, to transform arbitrary (but smooth) time-varying differential equations like (5) into their minimal controllability canonical state space from Goos and Pintelon (2016). The formulas are given explicitly in Section 4, rather than implicitly, like in Tóth (2010), and can also be applied to LPV differential equations like (6). As in Tóth (2010), we find that, in general, a minimal realization introduces dynamic dependence on the scheduling variable $p(t)$. Although it is not guaranteed that a static minimal state space model exists, we can start the optimization routine (Goos et al., 2014) from the obtained model. Proceeding in this way, a simple, static model approximation can be fitted.

Specifically, in a first step we will use the Linear Periodic Time-Varying (LPTV) IO identification method described in Section 3.4, and realize a corresponding LTV SS model in Section 4. Next, Section 5 establishes a link between this time-varying state space model and basis functions of the scheduling parameter $f\big(p(t)\big)$. In a final step, we optimize the model fit using simple basis functions that are suitable for control design. The complete workflow is depicted in Fig. 2. Section 6 illustrates the proposed approach on a simulation example, and discusses the properties of all modeling steps.