Automatica 71 (2016) 369–380

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Global swarming while preserving connectivity via Lagrange–Poincarè equations ☆

Aykut C. Satici [a], Mark W. Spong [b]

[a] Massachusetts Institute of Technology, Cambridge, MA, USA
[b] University of Texas at Dallas, Richardson, TX, USA

ARTICLE INFO

Article history:
Received 20 January 2015
Received in revised form
23 January 2016
Accepted 18 April 2016
Available online 22 June 2016

Keywords:
Lagrange–Poincaré equations
Multi-agent systems
Principal bundles
Horizontal–vertical
Swarming
Symmetry

ABSTRACT

In this paper, we exploit symmetry properties of multi-agent robot systems to design control laws that preserve connectivity while swarming. We start by designing a connectivity control law for agents with configuration spaces $\mathbb{R}^3$ and $SO(3)$ that is invariant under the action of the special Euclidean group $SE(3)$ and the special orthogonal group $SO(3)$, respectively. Therefore, the dynamics of such multi-agent systems is amenable to be reduced by these group actions. We then utilize the Lagrange–Poincaré equations that split the Euler–Lagrange equations for the multi-agent system into horizontal and vertical parts. The invariance of the connectivity controller implies that its control effort has zero vertical component. We then use the resulting vertical equations of motion to design a control law that asymptotically stabilizes the centroid and the orientation of the swarm at a desired pose.

1. Introduction

Mobile robot networks afford an inexpensive and robust method for achieving certain coverage tasks or cooperative missions. Many of the algorithms employed to achieve such tasks depend on communication between any two robots and hence require connectivity of the communication network. As a result, the problem of maintaining connectivity in mobile robot networks is an active area of research. For many applications, the edges or links in the mobile robot network are functions of the relative positions of nodes in the network. Thus, the connectivity of the network is affected by the motion of the robots, and the motion controllers must maintain connectivity in addition to achieving other goals.

One of the important goals of a multi-agent mobile network is coverage or surveillance of a given area. In particular, under this general heading there is a lot interest in tasks such as cooperative gradient following and formation control, search/exploration/rescue behavior. This requires the agents to swarm or move in formation along a desired path/trajectory. In other words, it is desired that the centroid of the formation moves along a specified desired trajectory. In addition, when avoiding contact with the environment is an issue, we may also want to specify a desired orientation trajectory of the multi-agent system.

A review of different methods to control and maintain connectivity can be found in Zavlanos, Egerstedt, and Pappas (2011). Connectivity can be maintained in a centralized or decentralized manner. One of the simplest methods of ensuring connectivity is to assume that the network is initially connected and that existing edges in the network are maintained for all time (Ji & Egerstedt, 2007; Zavlanos & Pappas, 2005). Another centralized method for maintaining connectivity in a group of mobile robots is to maximize the second smallest eigenvalue of the graph Laplacian (Kim & Mesbahi, 2006) when the edge strengths are non-increasing functions of the distance between robots. The resulting graph is always connected (Kim & Mesbahi, 2006). This method is effective for solving rendezvous problems, and can be extended to other applications (Zavlanos et al., 2011). Recently, in Sabattini, Secchi, Chopra, and Gasparri (2013) the authors have devised a control law which allows agents with first-order dynamics on $\mathbb{R}^n$ to perform a secondary task while their connectivity is preserved through a distributed control law that uses an estimation of the second smallest eigenvalue of the graph Laplacian. Defining distance constraints among a sufficiently many

☆ The material in this paper was partially presented at the 19th IFAC World Congress, August 24–29, 2014, Cape Town, South Africa. This paper was recommended for publication in revised form by Associate Editor Tamas Keviczky under the direction of Editor Christos G. Cassandras.

E-mail addresses: acsatici@mit.edu (A.C. Satici), mspong@utdallas.edu (M.W. Spong).

http://dx.doi.org/10.1016/j.automatica.2016.04.042
0005-1098/© 2016 Published by Elsevier Ltd.

agents and controlling the robots to achieve these distances yields the method of controlling the rigidity of a graph. This problem is still an active avenue of research. In Dimarogonas and Johansson (2008, 2009), the authors have shown that there is no gradient-based feedback control (centralized or decentralized) that achieves the goal of controlling the rigidity of a multi-agent robot network. The literature contains approaches (Oh & Ahn, 2010) where the control of the rigidity of a graph is achieved via a discontinuous state feedback.

In this paper, we consider the problem of simultaneously maintaining and regulating a connectivity measure of a multi-agent system while moving the agents as a "rigid-body" from an initial to a desired final configuration. The connectivity measure to be regulated may be taken either as the determinant of the reduced graph Laplacian matrix or the second smallest eigenvalue of the graph Laplacian matrix. In case the configuration space of the agents is $\mathbb{R}^3$, the behavior of the agents as a rigid body is captured as the pose of the simplex formed by the agents. When the configuration space of the agents is $SO(3)$, this behavior is captured by the configuration of one of the agents, in particular, we have used the Nth agent in this role in this work. The connectivity controller, which achieves and maintains connectivity, is applied as in Satici, Poonawala, Eckert, and Spong (submitted for publication). We show that this connectivity controller is invariant under the action of the relevant Lie group G. This becomes important when we introduce the splitting of the Euler–Lagrange equations into horizontal and vertical parts where the horizontal part governs the behavior of the multi-agent system's internal configurations, i.e., the position of the agents relative to each other while the vertical part governs the behavior of the agent formation as a rigid body. Since the connectivity control is invariant under the G-action the control effort expended by this controller has no vertical part.

After the splitting of the Euler–Lagrange equations into horizontal and vertical parts, we make use of the vertical part of the resulting equations to impose asymptotically convergent swarming behavior into the system. In other words, we use the available control input in the vertical direction to asymptotically stabilize the collectively desired trajectory of the multi-agent system. We provide simulation results where we take three agents each of whose configuration space is $\mathbb{R}^3$ or $SO(3)$ and show that they can be made to swarm with the desired collective motion while the connectivity measure is increased to a desired value.

Another contribution of this paper is that it presents a framework in which the swarming behavior of agents in $\mathbb{R}^3$ or $SO(3)$ can be cast into a canonical form globally. In other words, as opposed to Michael, Belta, and Kumar (2006), this work does not assume that the configuration space of the robots can be written as the product of a base space S and a group G. Although we use the coordinate form of the Lagrange–Poincaré equations, or the reduced Euler–Lagrange equations, our formulation actually holds globally. This means, one can potentially find a coordinate system whose domain of validity is larger that the one used in this paper and use this coordinate system to achieve the desired swarming behavior. The fact that inputs in charge of swarming behavior operate along the directions tangent to the group orbits means that any existing controller that acts in the horizontal space is unaffected provided the kinetic energy (metric) of the original system is group invariant. We exploit this property to employ our earlier connectivity controller to simultaneously achieve a desired connectivity measure.

2. Background

In this section, we introduce various definitions and background material that we will use in the sequel. We employ some of the standard notation and terminologies of mechanics and Lie groups, which may be found in Marsden and Ratiu (1999). We start by the definitions of geometric objects and specialize them for agents with configuration space $\mathbb{R}^3$ and $SO(3)$ in two subsections. We also discuss the connectivity controller potential function before the end of this section.

2.1. Preliminaries and definitions

Suppose that the configuration space of the multi-agent system that we are interested in is denoted by Q. In this work, we are primarily interested in the two cases $Q = \prod_{i=1}^N \mathbb{R}^3$ or $Q = \prod_{i=1}^N SO(3)$. We let a Lie group G act on the configuration space Q via a free and proper map $\Phi : G \times Q \to Q$.

If we let $\xi \in \mathfrak{g}$ to be an element of the Lie algebra of the Lie group G acting on the configuration space Q, then the **infinitesimal generator** $\xi_Q \in C^\infty(Q)$ of the group action is found by

$$\xi_Q(q) = \left.\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} \exp(\varepsilon\xi)\cdot q. \tag{1}$$

For each $q \in Q$, the **locked inertia tensor**, is the map $\mathbb{I}(q) : \mathfrak{g} \to \mathfrak{g}^*$, defined by

$$\langle \mathbb{I}(q)\eta, \zeta\rangle = \langle\!\langle \eta_Q(q), \zeta_Q(q)\rangle\!\rangle \tag{2}$$

where $\langle\!\langle \cdot, \cdot\rangle\!\rangle$ is the kinetic energy metric defined on TQ. The locked inertia tensor specifies the inertia of the system whose internal degrees of freedom are frozen. In other words, if the relative distances of the robots are constrained to be constant, then the locked inertia tensor is the inertia tensor of the resulting system.

Associated with the action of the Lie group G on Q is a momentum map $\mathbf{J} : TQ \to \mathfrak{g}^*$ defined by

$$\langle \mathbf{J}(q, \dot q), \xi\rangle = \langle\!\langle \dot q, \xi_Q(q)\rangle\!\rangle, \quad \forall \xi \in \mathfrak{g}. \tag{3}$$

The **mechanical connection** $A : TQ \to \mathfrak{g}$ is then found by the relation

$$A(q, \dot q) := \mathbb{I}(q)^{-1}\left(\mathbf{J}(q, \dot q)\right). \tag{4}$$

The **horizontal space** of the connection A is given by

$$\mathrm{hor}_q = \left\{(q, \dot q) \in T_qQ : \mathbf{J}(q, \dot q) = 0\right\} \tag{5}$$

which is the subspace of the tangent space at $q \in Q$ that is metric orthogonal to the orbits of the group action. The **vertical space** consists of vectors that are mapped to zero by the projection map $\pi : Q \to Q/G$

$$\mathrm{ver}_q = \left\{\xi_Q(q) \in T_qQ : \xi \in \mathfrak{g}\right\}. \tag{6}$$

These two subspaces of T_qQ are complementary, i.e., any a vector $(q, \dot q) \in T_qQ$ can be uniquely decomposed as

$$\dot q = \mathrm{hor}_q \dot q + \mathrm{ver}_q \dot q \tag{7}$$

where

$$\mathrm{ver}_q \dot q = \left[A(q, \dot q)\right]_Q(q) \quad \text{and} \quad \mathrm{hor}_q \dot q = \dot q - \mathrm{ver}_q \dot q. \tag{8}$$

2.1.1. Agents with configuration space $\mathbb{R}^3$

We first consider N agents, each of whose configuration q_i is an element of $\mathbb{R}^3$. These agents are all simple mechanical systems with kinetic energy $K_i(q_i, \dot q_i) = \frac{1}{2} m_i \|\dot q_i\|^2$, where $m_i > 0$ is the mass of the ith agent and $\dot q_i \in T_{q_i}\mathbb{R}^3 \cong \mathbb{R}^3$ is its velocity at the position q_i. The identification of the tangent space to $\mathbb{R}^3$ with $\mathbb{R}^3$ is a standard one and the interested reader is referred to do Carmo (1992) for more details. In this work, we are going to ignore the