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Brief paper

Distributed $\mathcal{L}_2$-gain output-feedback control of homogeneous and heterogeneous systems[☆]

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ABSTRACT

The importance of static output feedback (OPFB) design for aircraft control, process control, and elsewhere has been well documented since the 1960s, since full state measurements are not usually available in practical systems. This problem is compounded in the case of multi-agent systems (MAS) where each agent has its own state variable and measured outputs. Therefore, this paper addresses the $\mathcal{L}_2$-gain OPFB synchronization of linear MAS subject to external disturbances. Both homogeneous and heterogeneous MAS are considered. For homogeneous MAS, it is shown that the $\mathcal{L}_2$-gain static OPFB synchronization problem of MAS can be cast into the $\mathcal{L}_2$-gain static OPFB problem for a set of decoupled systems that depend on the graph topology. A modified Riccati equation is introduced which gives the OPFB gain and the coupling gain of the proposed static OPFB control protocol. For heterogeneous MAS, it is shown that the $\mathcal{L}_2$-gain synchronization problem can be cast into the $\mathcal{L}_2$-gain static OPFB problem of a set of decoupled systems plus a coupling condition on their dynamic compensators that depends on the graph topology. A certain novel gain matrix is introduced in the dynamics of the control protocol to improve the performance.

1. Introduction

Distributed control of multi-agent systems (MAS) on communication graphs has been well studied in the literature (Fax & Murray, 2004; Jadbabaie, Lin, & Morse, 2003; Lewis, Zhang, Hengster-Movric, & Das, 2014; Olfati-Saber & Murray, 2004; Qu, 2009; Ren & Beard, 2007), due to its potential applications in a variety of engineering systems. A control law, which depends only on the local neighbor information, is designed for each agent to make the network of agents converge to a common value of interest. If the common value that agents agree on is not prescribed, the problem is called regulation, and if all agents follow the trajectory of a leader node, the problem is known as leader–follower control (Hong, Hu, & Gao, 2006; Ren, Moore, & Chen, 2007).

The design of distributed control protocols for MAS with general linear dynamics has been considered in the literature (Li, Duan, Chen, & Huang, 2010; Ni & Cheng, 2010; Zhang, Lewis, & Das, 2011). However, attenuating the effect of the disturbance on the system performance is ignored in most existing methods. The disturbance rejection for MAS has been formulated as an H_∞ control problem in Lin, Jia, and Li (2008), Li, Duan, and Chen (2010) and Li, Duan, and Chen (2011) under a leaderless framework. Conditions in terms of linear matrix inequalities were given to ensure consensus with a desired H_∞ performance. Existing methods, however, mostly require the knowledge of the relative states between neighboring agents, which may not be available for measurement.

The importance of output feedback (OPFB) design, particularly static output feedback (Syrmos, Abdallah, Dorato, & Grigoriadis, 1997), for aircraft control (Stevens & Lewis, 2003), optimal process control (Lewis, Vrabie, & Syrmos, 2012), and elsewhere has been well documented since the 1960s, since full state measurements are not usually available in practical systems. This problem is compounded in the case of MAS where each agent has its own state

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variable and measured outputs. Therefore, this paper addresses the $\mathcal{L}_2$-gain OPFB synchronization of linear MAS subject to external disturbances.

To obviate the requirement for full state measurements, distributed observer-based OPFB and distributed dynamic OPFB protocols were derived in Liu and Jia (2010), Liu, Jia, Du, and Yuan (2009) and Zhao, Duan, Wen, and Chen (2012) for H_∞ control. For the leader–follower systems, distributed state-feedback protocols were designed in Liu and Lunze (2014) and Wen, Duan, Li, and Chen (2012), and a distributed observer-type protocol was presented in Hong, Chen, and Bushnell (2008). The above techniques assumed homogeneous MAS, that is, all agents have identical dynamics. The heterogeneous case, where agents can have different dynamics, has been considered by Lunze (2012), Su and Huang (2012) and Wieland, Sepulchre, and Allgöwer (2011). It is shown that each agent and its controller must contain an internal copy of the leader's dynamics. This work is based either on the output regulator equations (Huang, 2004; Knobloch, Isidori, & Flockerzi, 1993) or the idea of p-copy (Huang, 2004). Some works have been done on H_∞ control for heterogeneous systems (Dullerud & D'Andrea, 2004; Huang & Ye, 2014; Massioni, 2014; Peymani, Grip, Saberi, Wang, & Fossen, 2014; Scorletti & Duc, 2001).

Although several OPFB control protocols are designed for homogeneous MAS in the presence of disturbance, existing results are mainly dynamic OPFB methods. The static OPFB problem for single-agent systems is one of the most researched problems in the control society. The use of static OPFB allows flexibility and simplicity of implementation, and it is of extreme importance in practical controller design applications (Lewis et al., 2012; Stevens & Lewis, 2003). For disturbance-free case, decentralized static OPFB stabilization and synchronization of networks were studied in Menon and Edwards (2009). Static OPFB protocols are derived for consensus (Ma & Zhang, 2010) and synchronization of homogeneous MAS in the presence of disturbances (Hengster-Movric, Lewis, & Sebek, 2015). Moreover, for heterogeneous MAS, the design of a dynamic OPFB protocol has been considered for the case where the disturbances in the dynamics of the follower agents are generated by the leader.

In this paper, the distributed control of leader–follower problem subject to external disturbances using OPFB is considered. A complete approach to OPFB design is given for both homogeneous and heterogeneous MAS. For the homogeneous MAS, it is shown that this problem can be cast into the $\mathcal{L}_2$-gain static OPFB problem for a set of decoupled systems. A modified algebraic Riccati equation (ARE), which takes into account the spectrum property of the communication topology, is introduced. For the heterogeneous MAS, a modified dynamic OPFB control protocol is employed to solve the bounded $\mathcal{L}_2$-gain synchronization problem. It is shown that the $\mathcal{L}_2$-gain synchronization problem of heterogeneous MAS can be cast into the $\mathcal{L}_2$-gain static OPFB problem of a set of decoupled systems plus a coupling condition on their dynamic compensators that depends on the graph topology. It is also shown that the gains of the dynamic compensator can be found by solving AREs for each agent.

2. Preliminaries and problem formulation

Throughout this paper, the following notations are used: A matrix $T \in \mathbb{C}^{n\times n}$ is unitary if $T^*T = TT^* = I_n$. $\mathbf{1}_n \in \mathbb{R}^n$ denotes the vector whose elements are equal to 1. $\mathcal{L}_2[0,\infty)$ denotes the space of square integrable vector functions over $[0,\infty)$. $A\otimes B$ denotes the Kronecker product of matrices A and B. $diag(A_1,\ldots,A_n)$ represents a block diagonal matrix with matrices $A_i, i = 1,\ldots,n$, on its diagonal.

2.1. Graph theory

A *directed graph* $\mathcal{G}$ consists of a pair $(\mathcal{V},\mathcal{E})$, where $\mathcal{V} = \{\alpha_1,\ldots,\alpha_N\}$ is a finite nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is a set of ordered pairs of nodes, called edges. Denote the adjacency matrix as $\mathcal{A} = [a_{ij}]$ with $a_{ij} > 0$ if $(\alpha_i,\alpha_j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The set of nodes α_j with edges incoming to node α_i is called the neighbors of node i, namely $\mathcal{N}_i = \{\alpha_j : (\alpha_j,\alpha_i) \in \mathcal{E}\}$. The graph Laplacian matrix is defined as $L = D - \mathcal{A}$, which has all row sums equal to zero. $D = diag(d_i)$ is called the in-degree matrix, where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ is the weighted in-degree of node i. A (directed) tree is a connected digraph where every node except one, called the root, has in-degree equal to one. A graph has a spanning tree if a subset of the edges forms a directed tree.

Assumption 1. The digraph $\mathcal{G}$ contains a spanning tree and the leader is pinned to a root node.

2.2. $\mathcal{L}_2$-gain OPFB synchronization for homogeneous multi-agent systems

Consider N identical linear dynamic systems given by

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + D\omega_i \\ y_i &= Cx_i \quad i = 1,\ldots,N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^p$, $\omega_i(t) \in \mathbb{R}^m$ and $y_i(t) \in \mathbb{R}^q$ are the state, control input, external disturbance and output of node i, respectively. A, B, C and D are constant matrices with C full row rank. Let the leader dynamics be

$$\dot{x}_0 = Ax_0, \quad y_0 = Cx_0 \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ is leader state and $y_0(t) \in \mathbb{R}^q$ is the measured output.

Define the local neighborhood error of the node i as

$$\varepsilon_{y_i} = \sum_{j\in\mathcal{N}_i} a_{ij}(y_j - y_i) + g_i(y_0 - y_i) \tag{3}$$

where $g_i \geq 0$ is called the pinning gain, with $g_i > 0$ for a small subset of agents having direct access to the leader. Consider the static OPFB controller (Hengster-Movric et al., 2015) for each node i as

$$u_i = cK\varepsilon_{y_i} \tag{4}$$

where $c > 0$ is the coupling gain, and $K \in \mathbb{R}^{p\times q}$ is the feedback control gain matrix to be determined.

The synchronization error for agent i is defined as

$$\delta_i = x_i - x_0. \tag{5}$$

Let $x = [x_1^T,\ldots,x_N^T]^T$, $\underline{x}_0 = (\mathbf{1}_n \otimes I_n)x_0 \in \mathbb{R}^{nN}$, $\delta = [\delta_1^T,\ldots,\delta_N^T]^T$, $\omega = [\omega_1^T,\ldots,\omega_N^T]^T$ and $\varepsilon_y = [\varepsilon_{y1}^T,\ldots,\varepsilon_{yN}^T]^T$. Then the global synchronization dynamics become

$$\begin{aligned} \dot{\delta} &= (I_N\otimes A - c(L+G)\otimes BKC)\delta + (I_N\otimes D)\omega \\ \varepsilon_y &= -\big((L+G)\otimes C\big)\delta \end{aligned} \tag{6}$$

where $G = diag(g_1,\ldots,g_N)$ is the diagonal matrix of pinning gains. Since L has all row sums equal to zero, one has $L\underline{1} = 0$ with $\underline{1} = [1,\ldots,1]^T$. Moreover, under Assumption 1, $(L+G)$ is positive definite.

Problem 1 (*Bounded $\mathcal{L}_2$-gain Static OPFB Synchronization Problem for Homogeneous MAS*). Given N identical systems in (1) and a