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Robust control for a class of nonlinear systems with unknown measurement drifts[☆]

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ABSTRACT

This paper addresses the problem of designing a robust controller for a class of nonlinear systems whose states cannot be precisely measured caused by the *unknown* drifts in the powers of the measurement functions. By adopting the concept of homogeneity with monotone degrees and revamping the technique of adding a power integrator, a new design procedure is introduced to recursively construct a generalized homogeneous controller with monotone degrees as well as a Lyapunov function with unknown parameters. The proposed controller is able to globally stabilize a family of nonlinear systems with different measurement drifts whose bounds can be determined by solving an optimization problem.

1. Introduction

In this paper, we consider the global control problem for a class of nonlinear systems in the form

$$\begin{aligned}\dot{x}_i(t) &= x_{i+1}(t), \quad i = 1, \ldots, n-1,\\ \dot{x}_n(t) &= b(x(t))u(t) + a(x(t)),\\ y_j(t) &= x_j^{1+\varepsilon_j}(t), \quad j = 1, \ldots, n,\end{aligned} \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y_j(t)$'s are the system states, the control input and the measurements of system states, respectively. The uncertain nonlinear function $a(\cdot) : \mathbb{R}^n \to \mathbb{R}$ is a $\mathcal{C}^1$ function with $a(0) = 0$ and $b(\cdot) : \mathbb{R}^n \to \mathbb{R}$ is a continuous function. For $j = 1, \ldots, n$, the drift ε_j in the power of the measurement function $x_j^{1+\varepsilon_j}$ is an *unknown* ratio of an even integer and an odd integer.

In decades, the control problem for nonlinear systems has attracted a great deal of attention and many important results have been achieved (Dashkovskiy & Pavlichkov, 2012; He, Chen, & Yin, 2016; He, Dong, & Sun, 2016; Isidori, 1995; Praly, Andrea-Novel, & Coron, 1991; Tsinias, 1995a,b). For system (1), which represents an important class of nonlinear systems, namely feedback linearizable systems, the backstepping method (Kanellakopoulos, Kokotovic, & Morse, 1991) is known as an effective way for the state-feedback controller design, whose basic idea is to cancel the nonlinear terms via feedback. Therefore, this method requires the whole system to be fully known. Different from the backstepping method, the work (Lin & Qian, 2000; Qian & Lin, 2001) introduced the adding a power integrator technique, which relies on dominating, instead of cancelling the nonlinear terms. With this new tool, numerous stabilization results have been achieved for nonlinear systems with various structures and restrictions, for example, Back, Cheong, Shim, and Seo (2007), Fu, Ma, and Chai (2015), Zhai and Qian (2012) and the references therein.

It is demonstrated in Rosier (1992) that homogeneous systems inherit some nice properties from linear systems and provide us a new viewpoint to deal with the nonlinear systems (Kawski, 1990). However, the traditional definition for the homogeneous system with the uniformed degree can only encompass a small part of nonlinear systems. The recent work (Polendo & Qian, 2008) introduced a generalized homogeneity named homogeneity with monotone degrees (HWMD), which not only relaxes the restrictions on the uniformed homogeneity, but also leads to new controllers with better smoothness (Tian, Zhang, Qian, & Li, 2014; Zhang, Qian, & Li, 2013).

In practice, it is common that the sensors are not able to measure the system states accurately, e.g., the power drift ε_i in (1)

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may not be zero. For instance, as shown in Application Note for an Infrared (0000); Zhai and Qian (2012), the voltage output from an infrared distance sensor is a nonlinear function. A typical sensor for the real distance d only outputs d^p where the constant p is around 0.8 but its precise value is varying from products to products (Fig. 5 in Application Note for an Infrared (0000)). For example, if such a distance sensor is applied to the system of a particle moving under nonlinear viscous friction (Gong & Qian, 2007),

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = u - \text{sign}(x_2)|x_2|^{\alpha}, \quad 1 \le \alpha \le 5/3 \tag{2}$$

i.e., $y_1 = x_1^{1+\varepsilon_1}$ for an unknown constant ε_1, the stabilization problem becomes challenging. Motivated by this practical example, this paper aims to address the global control problem for a class of nonlinear systems with unknown measurement drifts. The main obstacles lie in two aspects: (i) With uncertain ε_i's, it is impossible to use the real information of system states x_i's to design the controller. (ii) The first derivative of the measurement function $y_1(x_1)$ does not exist at the point $x_1 = 0$ for a negative offset ε_1. This new obstacle cannot be handled by the method in Zhai and Qian (2012) where global stabilization is achieved under the condition that $\frac{\partial y_1(x_1)}{\partial x_1}$ is bounded.

Due to the aforementioned difficulties, the existing results cannot be used to solve the global stabilization problem for system (1). In this paper, we first identify the condition on the power drifts ε_i's such that the system is homogeneous with monotone degrees. Moreover, we provide a detailed selection process of the HWMD weights r_i's which can now contain unknown parameters. Then, by generalizing the adding a power integrator technique, a robust controller, which only utilizes the measurements of system states, is recursively constructed to render the closed-loop systems globally asymptotically stable.

2. Preliminaries and assumptions

The objective of this paper is to design a controller using the drifted measurement $y_i = x_i^{1+\varepsilon_i}$ for unknown ε_i's. To begin with, we revisit the recent definition of HWMD.

Definition 2.1 (*HWMD Polendo & Qian, 2008*). A continuous vector field $f(x) = [f_1(x), \ldots, f_n(x)]^T$, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$, is said to be homogeneous with monotone degrees, if there are positive real numbers $(r_1, \ldots, r_n)$ and $\tau_1 \ge \tau_2 \cdots \ge \tau_n > -r_n$, such that $\forall x \in \mathbb{R}^n \backslash \{0\}$, $\forall \epsilon > 0$, $f_j(\epsilon^{r_1}x_1, \ldots, \epsilon^{r_n}x_n) = \epsilon^{\tau_j + r_j} f_j(x)$, $j = 1, \ldots, n$. The constants r_i's and τ_i's are called homogeneous weights and degrees respectively.

Next we impose a condition on the power drifts ε_i's.

Definition 2.2. The power drift upper-bound $\varepsilon^*(n)$ for an n-dimensional system is chosen as $\varepsilon^*(1) = \varepsilon^*(2) = 1$ and when $n \ge 3$ the solution of the following problem.

Problem 2.1.

Max ε

$$\text{s.t. } a_{n-1} = 2, \qquad a_{i+1}(2 - a_i) = \left(\frac{1+\varepsilon}{1-\varepsilon}\right)^2,$$
$$a_i \ge 0, \ \varepsilon \le 1, \ 1 \le i \le n-2. \tag{3}$$

Solvability of Problem 2.1: Define $\Delta = \left(\frac{1+\varepsilon}{1-\varepsilon}\right)^2$ which increases along with ε from 0 to 1. The parameters a_i's can be represented as functions of Δ, i.e., $a_{n-1}(\Delta) = 2$, $a_i(\Delta) = 2 - \frac{\Delta}{a_{i+1}(\Delta)}$, when $a_{i+1} > 0$, $i = n-2, \ldots, 1$. It is clear that when $\Delta = 1$ or $\varepsilon = 0$, all a_i's in (3) are positive and by the continuity they will stay positive for a period of time. Noting that $a'_{n-2}(\Delta) = -\frac{1}{2} < 0$, one has, when $a_{n-2}(\Delta) > 0$, $a'_{n-3}(\Delta) = \frac{\Delta a'_{n-2} - a_{n-2}(\Delta)}{a^2_{n-2}(\Delta)} \le \frac{-1}{a_{n-2}(\Delta)} < -\frac{1}{2}$. By such analogy, it can be verified that, for $i = n-3, \ldots, 1$, when $a_{i+1}(\Delta) > 0$ and $a'_{i+1}(\Delta) < 0$

$$a'_i(\Delta) = \frac{\Delta a'_{i+1}(\Delta) - a_{i+1}(\Delta)}{a^2_{i+1}(\Delta)} \le \frac{-1}{a_{i+1}(\Delta)} < -\frac{1}{2}. \tag{4}$$

The relation (4) implies that $a_i(\Delta)$ will strictly decrease from $a_i(1) > 0$ until one of them crosses the zero line. Moreover we can show that $a_1(\Delta)$ will be the first one to hit the zero. Otherwise, assume that $a_k(\Delta)$ for a $k > 1$, arrives at zero first, then one has $\Delta = a_k(2 - a_{k-1}) = 0$, which implies that $a_{k+1} = \cdots = a_{n-1} = 0$. This contradicts to the assumption $a_{n-1} = 2$. Therefore, only $a_1(\Delta)$ is the first one that arrives at the zero and the corresponding ε reaches its maximum value ε^* when $a_1 = 0$. In other words, the following proposition holds:

Proposition 2.1. *For any* $1 \le \Delta < \left(\frac{1+\varepsilon^*}{1-\varepsilon^*}\right)^2$, *the corresponding* $a_i(\Delta) > 0, \forall 1 \le i \le n-1$.

Remark 2.1. In the following table, we list the value of ε^* for several cases by solving the optimization problem.

n	3	4	5	6	7	$\cdots$
ε^*	0.3333	0.1716	0.1056	0.0718	0.0521	$\cdots$

Assumption 2.1. There is a known constant $\bar{\varepsilon}$ such that the power drifts ε_i's satisfy $|\varepsilon_i| \le \bar{\varepsilon} < \varepsilon^*(n)$ where $\varepsilon^*(n)$ is the upper-bound given in Definition 2.2.

The next condition ensures that the system is controllable.

Assumption 2.2. There exists a positive constant b_0 such that $b(x(t)) \ge b_0 > 0$.

In what follows, we include two lemmas whose proofs can be found in Qian and Lin (2001).

Lemma 2.1. *For* $x \in \mathbb{R}$, $y \in \mathbb{R}$, $p \ge 1$

$$||x|^{\frac{1}{p}} \pm |y|^{\frac{1}{p}}| \le 2^{1-\frac{1}{p}} ||x| \pm |y||^{\frac{1}{p}}.$$

Lemma 2.2. *For constants* $c > 0$, $d > 0$ *and a continuous function* $\gamma(x, y) > 0$, *the following inequality holds:*

$$|x|^c |y|^d \le \frac{c}{c+d}\gamma(x, y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}(x, y)|y|^{c+d}.$$

3. Global stabilization of system (1)

First we show that we can choose appropriate homogeneous weights such that the system is HWMD.

Lemma 3.1. *Under Assumption 2.1, there exist constants* $m_i > 0$, $1 \le i \le n$ *such that under the weights* $r_i = \frac{1}{m_i(1+\varepsilon_i)}$, $1 \le i \le n$, *and* $r_{n+1} = 1$, *the following holds* $\tau_1 \ge \tau_2 \ge \cdots \ge \tau_n$, *with* $\tau_i = r_{i+1} - r_i$, $1 \le i \le n$.

Proof. Based on $\bar{\varepsilon}$ which is strictly less than $\varepsilon^*(n) \le 1$ as stated in Assumption 2.1, $\bar{a}_i$ can be fixed as

$$\bar{a}_{n-1} = 2, \qquad \bar{a}_i = 2 - \left(\frac{1+\bar{\varepsilon}}{1-\bar{\varepsilon}}\right)^2 \frac{1}{\bar{a}_{i+1}}, \quad i = n-2, \ldots, 1. \tag{5}$$

By Proposition 2.1, $\bar{a}_i$'s in (5) apparently are strictly positive since they have not reached the zero yet.