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Finite-time connectivity preservation rendezvous with disturbance rejection☆

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ABSTRACT

This paper introduces a discontinuous rendezvous algorithm for multi-agent systems which allows a group of agents to achieve finite-time rendezvous with connectivity preservation as well as disturbance rejection. The proposed discontinuous rendezvous algorithm is based on a class of general communication functions with certain available information range. Nonsmooth stability analysis and graph theory are employed to obtain the finite-time rendezvous. An upper bound on the convergence time is also given. Our result simultaneously satisfies the three features: connectivity preservation, disturbance rejection and finite-time convergence.

1. Introduction

In recent years, the study of distributed control of multi-agent systems has attracted a great deal of attention from the control community. For more details, we refer the reader to the recent survey paper Cao, Yu, Ren, and Chen (2013) and references therein. Indeed, multi-agent systems have found applications in control engineering including multi-vehicle systems, mobile sensor networks and cooperative robot systems (Chung & Slotine, 2009; Lynch, Schwartz, Yang, & Freeman, 2008; Murray, 2007). The design of distributed control algorithms is an important issue in the coordination of multi-agent systems. Our focus in this paper is on algorithms for the rendezvous problem. Rendezvous is the motion in which a large number of agents, using only limited information and simple rules, reach a location simultaneously.

Communication between agents is usually characterized by the interaction topology. Due to limited information range and communication capabilities, the interaction topology may change over time. A basic assumption made in the literature on rendezvous algorithms is that the interaction topology can remain connected frequently enough during the whole evolution (Cortés, Martínez, & Bullo, 2006; Dimarogonas & Kyriakopoulos, 2007; Fan, Feng, Wang, & Qiu, 2011; Lin, Morse, & Anderson, 2007; Song, Feng, Wang, & Fan, 2012). However, it is always difficult to verify this assumption in practice. A way of overcoming this important issue is to preserve the connectivity of the interaction topology. Some connectivity preservation rendezvous algorithms have appeared in the literature, see, e.g., Gustavi, Dimarogonas, Egerstedt, and Hu (2010), Ji and Egerstedt (2007), Xiao, Wang, and Chen (2012) and Su, Wang, and Chen (2010). The authors in Dong and Huang (2014) further studied the connectivity preservation rendezvous problem subject to external disturbance. Note that those protocols reach rendezvous over an infinite-time horizon with exponential convergence. In many practical applications, it is of particular interest to achieve fast convergence. It is a desirable property that cooperative systems enjoy finite-time stability Bhat and Bernstein (2000). Recently, Hui (2011) proposed a discontinuous rendezvous algorithm for studying the finite-time connectivity preservation rendezvous problem of multi-agent systems with available information range. The available information range is defined by a specific communication function.

For finite-time rendezvous with disturbance rejection, discontinuous algorithms are needed to achieve finite-time convergence in the presence of external disturbance. Very recently, the authors in Franceschelli, Pisano, Giua, and Usai (2015) proposed a discontinuous algorithm to establish finite-time convergence with disturbance rejection under static interaction topology. However, to the best of our knowledge, there have been rather few studies on finite-time rendezvous with connectivity preservation and disturbance rejection.

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Motivated by the above discussions, we investigate the finite-time connectivity preservation rendezvous problem for multi-agent system in the presence of external disturbance. We introduce a class of general discontinuous communication functions which contains the specific communication function used in Hui (2011) as a special case. Based on non-smooth stability analysis and graph theory, finite-time rendezvous is derived and an upper bound on the convergence time is also given. The main contribution of this paper is to provide a result that simultaneously satisfies the three features: connectivity preservation, disturbance rejection and finite-time convergence.

The rest of this paper is organized as follows. We introduce the mathematical preliminaries on graph theory and nonsmooth stability theory in Section 2. The finite-time rendezvous problem is formulated in Section 3. Section 4 provides finite-time convergence result, which is illustrated by an example in Section 5. We give a conclusion in Section 6.

2. Mathematical preliminaries

2.1. Graph theory

In this paper, we consider the undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of agents and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. The neighbor set of agent i is defined as $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$. For the undirected graph $\mathcal{G}$, we have $j \in \mathcal{N}_i$ if and only if $i \in \mathcal{N}_j$. In this case, we call agents i and j are neighboring. A path from i to j is a sequence of distinct agents, starting from i and ending with j, such that each pair of consecutive agents is neighboring. If there is a path from i to j, then they are said to be connected. If all pairs of agents in $\mathcal{G}$ are connected, then $\mathcal{G}$ is said to be connected.

2.2. Discontinuous differential equations

Consider the differential equation given by

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0, \ t \geq 0 \tag{1}$$

where $f : \mathbb{R}^d \to \mathbb{R}^d$ is measurable and essentially locally bounded (Filippov, 1988). The *Filippov solution* of (1) is defined by an absolutely continuous solution $x : [0, \tau] \to \mathbb{R}^d$ such that

$\dot{x}(t) \in \mathcal{K}[f](x(t))$ for almost all $t \in [0, \tau]$.

The *Filippov set-valued map* $\mathcal{K}[f] : \mathbb{R}^d \to 2^{\mathbb{R}^d}$ is defined by

$$\mathcal{K}[f](x) \triangleq \bigcap_{\delta>0} \bigcap_{\mu(S)=0} \overline{\text{co}}\{f(B_\delta(x) \backslash S)\}, \tag{2}$$

where we denote by μ the Lebesgue measure in $\mathbb{R}^d$, by "$\overline{\text{co}}$" the convex closure, and by $B_\delta(x)$ the open ball centered at x with radius δ. It follows from Paden and Sastry (1987, Theorem 1) that there exists a set $\mathcal{N}_f \subset \mathbb{R}^d$, $\mu(\mathcal{N}_f) = 0$ such that for every $\mathcal{W} \subset \mathbb{R}^d$, $\mu(\mathcal{W}) = 0$

$$\mathcal{K}[f](x) = \overline{\text{co}} \left\{ \lim_{i\to\infty} f(x_i) : x_i \to x, \ x_i \notin \mathcal{N}_f \cup \mathcal{W} \right\}.$$

The Filippov set-valued map (2) is upper semi-continuous with nonempty, compact, convex values and locally bounded, from which the existence of Filippov solution of (1) follows Filippov (1988). A *maximal solution* is a Filippov solution whose domain of existence is maximal, i.e., cannot be extended any further. A set M is *weakly invariant* (resp., *strongly invariant*) with respect to (1) if for every $x_0 \in M$, M contains a maximal solution (resp., all maximal solutions) of (1).

To establish the non-smooth stability theory of (1), we need the following notions for a locally Lipschitz continuous function $V : \mathbb{R}^d \to \mathbb{R}$. We use the notation $\partial V(x)$ to denote the *generalized gradient* of V at x, $\mathcal{L}_f V(x)$ to denote the *set-valued Lie derivative* of V with respect to f at x, and $\max \mathcal{L}_f V(x)$ to denote the largest element of nonempty set $\mathcal{L}_f V(x)$. Due to space limitation, we do not recall these notions and definitions. See, e.g., Bacciotti and Ceragioli (1999), Clarke (1983) and Cortés and Bullo (2005) for the details.

3. Problem statement

Consider a group of N agents with the dynamics of each agent described by the single integrator

$$\dot{x}_i(t) = d_i(t, x) + u_i(t), \quad x_i(0) = x_{i0}, \ t \geq 0 \tag{3}$$

where for $1 \leq i \leq N$, $x_i(t) \in \mathbb{R}$ denotes the information state of agent i, $d_i(t, x)$ is the external disturbance of agent i and $u_i(t)$ is the control input to be designed. Note that the term $d_i(t, x)$ can also be understood as couplings or perturbations. We make the following assumption.

Assumption 1. $d_i(t, x)$ is continuous and bounded with respect to t and x, i.e., $|d_i(t, x)| \leq \lambda$ for all $x \in \mathbb{R}^N$ and $t \geq 0$, where λ is a positive constant.

The connectivity among agents is modeled by an undirected graph $\mathcal{G}$. Communication links among agents are often unreliable due to available information range. Link failures or creations always lead to dynamics information exchange topologies. Here, by available information range, we mean that the information is not available for feedback beyond certain range among agents while becomes available for feedback when agents are within this range (Hui, 2011). To achieve an agreement on a location among agents, for agent i, we propose the following control law:

$$u_i(t) = \sum_{j\in\mathcal{N}_i} \phi(|x_i - x_j|)\text{sgn}(x_j - x_i) \tag{4}$$

where $\text{sgn}(\cdot)$ is the standard signum function and $\mathcal{N}_i$ is the neighbor set of agent i in $\mathcal{G}$. The function $\phi : [0, \infty) \to [0, \infty)$ satisfies the following assumptions:

(A_1) ϕ is continuous except at point $r > 0$.
(A_2) There exists a positive constant α such that $\phi(s) \geq \alpha$ for $0 \leq s < r$ and $\phi(s) = 0$ for all $s \geq r$.
(A_3) $\int_0^r \phi(s)ds = \infty$.

Remark 1. The constant r can be regarded as the sensing radius of the agents. A class of general communication functions is involved in the protocol (4). A concrete example is the function used in Hui (2011). Another class of examples for ϕ is given by

$$\phi(s) = \begin{cases} \kappa/(r-s)^q, & 0 \leq s < r \\ 0, & s \geq r \end{cases}$$

where $\kappa > 0$ and $q \geq 1$.

Let $\Omega = \{x = (x_1 \ldots, x_N)^T \in \mathbb{R}^N : |x_i - x_j| < r, \text{ for all } (i,j) \in \mathcal{E}\}$. Define the nonnegative function $V(x) : \Omega \to [0, \infty)$ to be

$$V(x) = \sum_{(i,j)\in\mathcal{E}} \int_0^{|x_i-x_j|} \phi(s)ds. \tag{5}$$

Lemma 1. *Consider the closed-loop system given by (3) and (4). Let Assumption 1 hold. Then the set Ω is invariant for the system, i.e., for the initial data $x_0 = (x_{10}, \ldots, x_{N0})^T \in \Omega$, the solution $x(t) = (x_1(t), \ldots, x_N(t))^T$ remains in Ω for all $t \geq 0$.*