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Signal separation from X-ray image sequence using singular value decomposition

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ABSTRACT

This work proposes singular value decomposition (SVD) to separate the signal from a noisy X-ray image sequence without any *prior* knowledge of the noise. SVD is based on the theory that the noise is always uncorrelated to the signal in a noisy image, and SVD, which belongs to Blind Source Separation (BSS), can decorrelate the signal from the noise components. To apply this proposed denoising method, two groups of X-ray images produced at 25 kV & 20 mAs and 34 kV & 20 mAs are sampled. To measure the proposed denoising method, ROIs with differing glandularity are selected. This work supports the use of SVD in X-ray image denoising. Normally, the separated signal will be less noisy when more noisy images are included for separating signals. Compared with other classical denoising methods, SVD is superior in reducing noise and improving CNR or SNR.

1. Introduction

X-ray imaging makes it possible to clearly view an object's interior, and thus, this technique is important for the advancement of science and technology. Noise always exists in images, including X-ray images. According to the observations of X-ray images, noise can obscure an object's details. In a well-designed X-ray imaging system, there are many kinds of noise, including but not limited to electronic noise, structural noise, and quantum noise; however, the dominant noise source follows a compound Poisson distribution [1].

Most noise reduction methods smooth the information [2,3]. At present, noise modeling [4–10] and frame averaging (FA) [11,12] are two good methods to reduce the X-ray image's noise. As to noise modeling, for example, after the X-ray image is transformed to independent component analysis (ICA) domain, the noise is removed by shrinkage related to the Poisson noise distribution [13]. FA is the common denoising method for the medical images [14] and works on the assumption that the noise is truly random and that random fluctuations above and below actual image data will gradually even out as one averages more images. FA can increase the signal-to-noise ratio (SNR) if the imaged object remains relatively static.

In an image, the noise is always said to be uncorrelated to the signal (based on Blind Source Separation (BSS) theory). Therefore, second-order statistical (SOS) SVD can decorrelate the noise and signal. BSS is a statistical tool for analyzing multidimensional data, and, in recent years, it has been applied to image fusion, image enhancement, feature extraction, artifact removal, signal demixing, image separation and scattered ray reduction. Meanwhile, SVD is a factorization of a real or complex matrix in linear algebra. Formally, SVD of an $m \times n$ real or complex matrix M is a factorization of the form $M = U\Sigma V^T$, where the columns of U are the left singular vectors, Σ is an $m \times n$ rectangular diagonal matrix with nonnegative real numbers on the diagonals, and V^T has rows that are the right singular vectors. Before, SVD has removed the image noise by setting a threshold to abandon the singular vectors of small singular values [15,16].

In this research, based on BSS theory, we use SVD to remove the noise from an X-ray image sequence. This proposed method is denoted as BSS SVD. In an X-ray image sequence, any frame is taken as a combination of one stable signal and a lot of random noise. To apply this proposed denoising method, at least two frames must be sampled to constitute an image sequence. Denoising performance will be more accurate when more frames are included in an image sequence. However, in order to save time in the following experiment, we set the frame amount of an image sequence to vary from one to eleven. To evaluate the denoised images, we calculate terms such as the difference signal, mean value (MV), standard deviation

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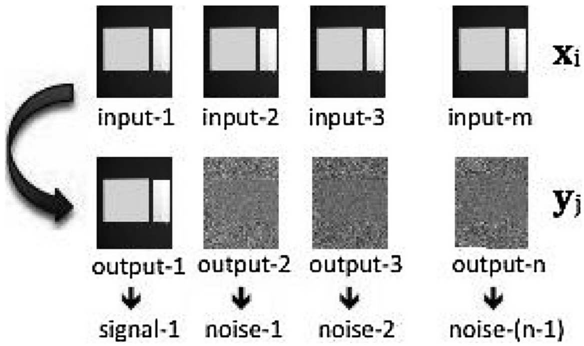

Fig. 1. Signal separating from an image sequence.

(SD), contrast-to-noise ratio (CNR), SNR and weighting factor *w* for dual-energy imaging.

2. Method

Any noisy image is composed of useful signal and unwanted noise. Moreover, the signal is static while the noise is usually random. As in the upper row in Fig. 1, such an X-ray image sequence can be denoted as matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m]^T$, and x_i is any linear combination of several noise components and one signal component, which is given in Eq. (1).

$$x_i = \sum\nolimits_{j=1}^{n} a_{ij} s_j, \tag{1}$$

where a_{ij} is the coefficient, and $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_n]^T$ is the source matrix including one signal component and *n*-1 noise components. *m* is the number of observation vectors x_i, and *n* is the number of source components. Notably, BSS (Blind Source Separation) theory suggests that $2 \le n \le m$. However, in this research, $2 \le n = m$.

To get the source s_j, the separation matrix W is necessary to decorrelate the observation **X** by using the statistical method. **Y**, an approximating matrix of **S**, will be attained, which is described as in Eq. (2).

$$Y = WX = WAS \approx A^{-1}AS = S, \tag{2}$$

where $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n]^T$ is the matrix composed of the estimated source $\mathbf{y}_j$, which is close to $\mathbf{s}_j$. As in Fig. 1, the bottom row is the output **Y,** which includes one denoised signal and several noise components.

The algorithm flow dictates that after an SVD prewhitening procedure of the original images, an asymmetric SVD is applied to process a single time-delayed covariance matrix of the whitened image data. The summarized steps are given below [17].

Step 1: Estimate the covariance matrix $R_X = E(\mathbf{XX}^T)$;

Step 2: Compute the SVD of R_X and estimate the number of sources, noise variance σ and singular values Ψ_1, Ψ_2, $\Psi_3\ldots$

Step 3: Data transformation: $\mathbf{X'} = C\mathbf{X}$ and $C = \mathrm{diag}(1/\Psi_1, 1/\Psi_2, 1/\Psi_3, \ldots)$

Step 4: Select a 'τ' and estimate $R_{X'}(\tau) = E(\mathbf{X'}(t)\,\mathbf{X'}(t-\tau)^T)$

Step 5: Compute the SVD of $(R_{X'}(\tau) + R_{X'}(\tau)^T)/2$ and **V** is the singular vector

Step 6: Estimate source is $Y = \mathbf{V}^T C\mathbf{X}$, $W = V^T C$

3. Experimental results and analysis

In this research, two X-ray image sequences of size 1914 × 2294 are selected. the sequences are grouped based on their production at a dual-energy of 25 kV & 20 mAs and 34 kV & 20 mAs. More frames included in an image sequence would provide better accuracy. However, here, 11 frames are sampled in order to save time.

In Fig. 2, the right cubic block is the phantom that is used for this denoising analysis. The right cubic block is usually used for analyzing breast imaging. As shown in Fig. 2(a), we analyze five differing compositions of adipose and glandular material, in which the glandular ratio is 100%, 75%, 50%, 25% and 0% from top to bottom. In Fig. 2(b), as indicated by the green line, the left half part of this phantom is the corresponding region of interest (ROI) that includes five sub-regions marked from 1 to 5.

Fig. 3 shows the noise degree before and after applying BSS SVD. MV represents signal and SD represents noise. Fig. 3(a) shows the original noise before denoising, and Fig. 3(b) shows the residual noise after denoising. In Fig. 3(b), the dotted lines denote BSS SVD denoising and real lines denote FA denoising. Five colors represent the data from five different sub-ROIs. In each sub-ROI prior to denoising, MV is steady while SD fluctuates a little, which is normal for the existence of random noise. After denoising, MV remains almost the same while SD gradually declines as more noisy images are included. To measure the residual noise in the denoised images, the difference image between the two denoised images was obtained by assuming that the noise in the denoised images is equal to $2^{1/2}$ of the noise in the difference image. It is obvious that the images produced at 34 kV & 20 mAs are less noisy than those produced at 25 kV & 20 mAs. BSS SVD is superior to FA in noise

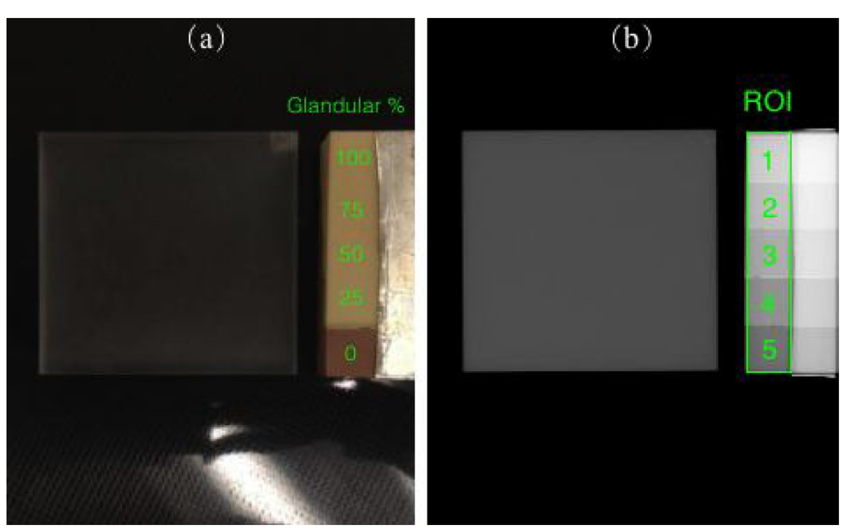

Fig. 2. The breast material phantom. (a). The visible image; (b). The X-ray image.