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Economic model predictive control without terminal constraints for optimal periodic behavior ☆

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ABSTRACT

In this paper, we analyze economic model predictive control schemes without terminal constraints, where the optimal operating behavior is not steady-state operation, but periodic behavior. We first show by means of a counterexample, that a classical receding horizon control scheme does *not* necessarily result in an optimal closed-loop behavior. Instead, a multi-step MPC scheme may be needed in order to establish near optimal performance of the closed-loop system. This behavior is analyzed in detail, and we show that under suitable dissipativity and controllability conditions, desired closed-loop performance guarantees as well as convergence to the optimal periodic orbit can be established.

1. Introduction

In recent years, the study of economic model predictive control (MPC) schemes has received a significant amount of attention. In contrast to standard stabilizing MPC, the control objective is the minimization of some general performance criterion, which need not be related to any specific steady-state to be stabilized. These type of control problems arise in many different fields of application, ranging, e.g., from the process industry over building climate control or the control of wind turbines to the development of sustainable climate policies (see, e.g., Amrit, Rawlings, & Biegler, 2013; Chu, Duncan, Papachristodoulou, & Hepburn, 2012; Gros, 2013; Heidarinejad, Liu, & Christofides, 2012; Hovgaard, Larsen, Edlund, & Jørgensen, 2012; Mendoza-Serrano & Chmielewski, 2014). In the literature, various different economic MPC schemes have been developed for which desired closed-loop properties such as performance estimates or convergence can be guaranteed. These include schemes with additional terminal equality or terminal region constraints (Amrit, Rawlings, & Angeli, 2011; Angeli, Amrit, & Rawlings, 2012), with generalized terminal constraints (Fagiano & Teel, 2013; Müller, Angeli, & Allgöwer, 2013), without terminal constraints (Grüne, 2013), and Lyapunov-based schemes (Heidarinejad et al., 2012) (see also the recent survey article Ellis, Durand, & Christofides, 2014).

A distinctive feature of economic MPC is the fact that the closed-loop trajectories are not necessarily convergent to a steady-state, but can exhibit more complex, e.g., periodic, behavior. In particular, the optimal operating behavior for a given system depends on its dynamics, the considered performance criterion and the constraints which need to be satisfied. The case where steady-state operation is optimal is by now fairly well understood, and various closed-loop guarantees have been established in this case. For example, a certain dissipativity property is both sufficient (Angeli et al., 2012) and (under a mild controllability condition) necessary (Müller, Angeli, & Allgöwer, 2015) for a system to be optimally operated at steady-state. The same dissipativity condition (strengthened to strict dissipativity) was used in Amrit et al. (2011) and Angeli et al. (2012) to prove asymptotic stability of the optimal steady-state for the resulting closed-loop system with the help of suitable terminal constraints. Similar (practical) stability results were established in Grüne (2013) and Grüne and Stieler (2014) without such terminal constraints.

On the other hand, the picture is still much less complete in case that the optimal operating behavior is non-stationary. This situation occurs in many cases of practical interest, such as in certain chemical reactors (see, e.g. Amrit et al., 2013; Angeli et al., 2012; Ellis et al., 2014) or in applications with time varying (energy) prices or demand (see, e.g., Limon, Pereira,

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Muñoz de la Peña, Alamo, & Grosso, 2014; Mendoza-Serrano & Chmielewski, 2014). For such cases, in Angeli et al. (2012) it was shown that when using some periodic orbit as (periodic) terminal constraint within the economic MPC problem formulation, then the resulting closed-loop system will have an asymptotic average performance which is at least as good as the average cost of the periodic orbit. Convergence to the optimal periodic orbit was established in[1] Huang et al. (2011) and Zanon, Gros, and Diehl (2013) using similar terminal constraints, and in Limon et al. (2014) for linear systems and convex cost functions using less restrictive generalized periodic terminal constraints. Furthermore, dissipativity conditions which are suited as sufficient conditions such that the optimal operating behavior of a system is some periodic orbit were recently proposed in Grüne and Zanon (2014).

In this paper, we consider economic MPC without terminal constraints for the case where periodic operation is optimal. Using no terminal constraints is in particular desirable in this case as the optimal periodic orbit then need not be known a priori (i.e., for implementing the economic MPC scheme). Furthermore, the online computational burden might be lower and a larger feasible region is in general obtained. We first show by means of a counterexample (see Section 3), that the classical receding horizon control scheme, consisting of applying the first step of the optimal predicted input sequence to the system at each time, does *not* necessarily result in an optimal closed-loop performance. We then prove in Section 4 that this undesirable behavior can be resolved by possibly using a multi-step MPC scheme instead. In particular, we show that the resulting closed-loop system has an asymptotic average performance which is equal to the average cost of the optimal periodic orbit (up to an error term which vanishes as the prediction horizon increases). This recovers the results of Angeli et al. (2012), where periodic terminal constraints were used as discussed above. In Section 5 we derive checkable sufficient conditions based on dissipativity and controllability in order to apply the results of Section 4. Furthermore, in Section 6 we show that under the same conditions, also (practical) convergence of the closed-loop system to the optimal periodic orbit can be established.

We close this section by noting that our analysis partly builds on the one in Grüne (2013), where closed-loop performance guarantees and convergence results for economic MPC without terminal constraints were established for the case where the optimal operating behavior is steady-state operation. However, while some of the employed concepts and ideas are similar to those in Grüne (2013), various properties of predicted and closed-loop sequences are different in the periodic case considered in this paper, and hence also different analysis methods are required. Finally, we note that a preliminary version of some of the results of this paper have appeared in the conference paper (Müller & Grüne, 2015). Compared to Müller and Grüne (2015), the main novelties of the present paper are a more comprehensive exposition of the subject including various additional remarks and examples as well as all proofs (which were partly missing in Müller and Grüne (2015)), the development of our results using a more general dissipativity condition, and the establishment of closed-loop (practical) convergence to the optimal periodic orbit.

2. Preliminaries and setup

Let $\mathbb{I}_{[a,b]}$ denote the set of integers in the interval $[a, b] \subseteq \mathbb{R}$, and $\mathbb{I}_{\geq a}$ the set of integers greater than or equal to a. For a set $\mathcal{A} \subseteq \mathbb{I}_{\geq 0}$, $\#\mathcal{A}$ denotes its cardinality (i.e., the number of elements). For $a \in \mathbb{R}$, $\lfloor a \rfloor$ is defined as the largest integer smaller than or equal to a. The distance of a point $x \in \mathbb{R}^n$ to a set $\mathcal{A} \subseteq \mathbb{R}^n$ is defined as $|x|_{\mathcal{A}} := \inf_{a\in\mathcal{A}} |x - a|$. For a set $\mathcal{A} \subseteq \mathbb{R}^n$ and $\varepsilon > 0$, denote by $B_\varepsilon(\mathcal{A}) := \{x \in \mathbb{R}^n : |x|_{\mathcal{A}} \leq \varepsilon\}$. By $\mathcal{L}$ we denote the set of functions $\varphi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ which are continuous, nonincreasing and satisfy $\lim_{k\to\infty} \varphi(k) = 0$. Furthermore, by $\overline{\mathcal{KL}}$ we denote the set of functions $\gamma : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for each $\varphi \in \mathcal{L}$, the function $\overline{\gamma}(k) := \gamma(\varphi(k), k)$ satisfies $\overline{\gamma} \in \mathcal{L}$. Note that the definition of a $\overline{\mathcal{KL}}$-function requires weaker properties than those for classical $\mathcal{KL}$-functions, i.e., each $\mathcal{KL}$-function is also a $\overline{\mathcal{KL}}$-function (but the converse does not hold).

We consider nonlinear discrete-time systems of the form

$$x(k+1) = f(x(k), u(k)), \quad x(0) = x \tag{1}$$

with $k \in \mathbb{I}_{\geq 0}$ and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. System (1) is subject to pointwise-in-time state and input constraints $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ and $u(k) \in \mathbb{U} \subseteq \mathbb{R}^m$ for all $k \in \mathbb{I}_{\geq 0}$. For a given control sequence $u = (u(0), \dots, u(K)) \in \mathbb{U}^{K+1}$ (or $u = (u(0), \dots) \in \mathbb{U}^\infty$), denote by $x_u(k, x)$ the corresponding solution of system (1) with initial condition $x_u(0, x) = x$. For a given $x \in \mathbb{X}$, the set of all feasible control sequences of length N is denoted by $\mathbb{U}^N(x)$, where a feasible control sequence is such that $u(k) \in \mathbb{U}$ for all $k \in \mathbb{I}_{[0,N-1]}$ and $x_u(k, x) \in \mathbb{X}$ for all $k \in \mathbb{I}_{[0,N]}$. Similarly, the set of all feasible control sequences of infinite length is denoted by $\mathbb{U}^\infty(x)$. In the following, we assume for simplicity that $\mathbb{U}^\infty(x) \neq \emptyset$ for all $x \in \mathbb{X}$. This means that for all initial conditions $x \in \mathbb{X}$, there exists a trajectory which stays in $\mathbb{X}$ for all times, i.e., the set $\mathbb{X}$ is control invariant under controls in $\mathbb{U}$. This assumption might be restrictive in general, and it can be relaxed if desired, using, e.g., methods similar to those in Grüne and Pannek (2011, Chap. 8) or Faulwasser and Bonvin (2015). However, the technical details of such an extension are beyond the scope of this paper.

Remark 1. For ease of presentation, we use decoupled state and input constraint sets $\mathbb{X}$ and $\mathbb{U}$ in the statement of our results. Nevertheless, all results in this paper are also valid for possibly coupled state and input constraints, i.e., $(x(k), u(k)) \in \mathbb{Z}$ for all $k \in \mathbb{I}_{\geq 0}$ and some $\mathbb{Z} \subseteq \mathbb{R}^n \times \mathbb{R}^m$, which will also be used in the examples. □

System (1) is equipped with a stage cost function $\ell : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$, which is assumed to be bounded from below on $\mathbb{X} \times \mathbb{U}$, i.e., $\ell_{\min} := \inf_{x\in\mathbb{X},u\in\mathbb{U}} \ell(x, u)$ is finite. Note that this is, e.g., the case if $\mathbb{X} \times \mathbb{U}$ is compact and ℓ is continuous. Without loss of generality, in the following we assume that $\ell_{\min} \geq 0$. We then define the following finite horizon cost functional

$$J_N(x, u) := \sum_{k=0}^{N-1} \ell(x_u(k, x), u(k)) \tag{2}$$

and the corresponding optimal value function

$$V_N(x) := \inf_{u\in\mathbb{U}^N(x)} J_N(x, u). \tag{3}$$

In the following, we assume that for each $x \in \mathbb{X}$, a control sequence $u^*_{N,x} \in \mathbb{U}^N(x)$ exists such that the infimum in (3) is attained, i.e., $u^*_{N,x}$ satisfies $V_N(x) = J_N(x, u^*_{N,x})$. Since we assume that $\mathbb{U}^\infty(x) \neq \emptyset$ for all $x \in \mathbb{X}$, this is, e.g., satisfied if f and ℓ are continuous and $\mathbb{U}$ is compact. A standard MPC scheme without additional terminal cost and terminal constraints then consists of minimizing, at each time instant $k \in \mathbb{I}_{\geq 0}$ with current system state $x = x(k)$, the cost functional (2) with respect to $u \in \mathbb{U}^N(x)$ and applying the first element of the resulting optimal input sequence $u^*_{N,x}$ to the system. This means that the resulting receding horizon control input to system (1) is given by $u_{MPC}(k) := u^*_{N,x_{u_{MPC}}(k,x)}(0)$, where $x_{u_{MPC}}(\cdot, x)$ denotes the corresponding closed-loop state sequence.

[1] In Huang, Harinath, and Biegler (2011), however, again the standard assumption as in stabilizing MPC, i.e., positive definiteness of the cost function, was imposed, which means that no general performance criterion as in economic MPC could be considered.