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Observation of nonlinear systems via finite capacity channels: Constructive data rate limits ☆

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ABSTRACT

The paper deals with observation of nonlinear and deterministic, though maybe chaotic, discrete-time systems via finite capacity communication channels. We introduce several minimum data-rate limits associated with various types of observability, and offer new tractable analytical techniques for their both upper and lower estimation. Whereas the lower estimate is obtained by following the lines of the Lyapunov's linearization method, the proposed upper estimation technique is along the lines of the second Lyapunov approach. As an illustrative example, the potential of the presented results is demonstrated for the system which describes a ball vertically bouncing on a sinusoidally vibrating table. For this system, we provide an analytical computation of a closed-form expression for the threshold that separates the channel data rates for which reliable observation is and is not possible, respectively. Another illustration is concerned with the celebrated Hénon system. The offered sufficient data rate bound is accompanied with a constructive observer that works whenever the channel capacity fits this bound.

1. Introduction

Recent advances in communication technology have created the possibility of large-scale control systems, where the control tasks are distributed over many agents orchestrated via a communication network; a particular example can be found in modern industrial systems, where the components are often connected over digital band-limited serial communication channels (Gao, Chen, & Lam, 2008; Liu et al., 2014; Postoyan, van de Wouw, Nešić, & Heemels, 2014; Wang, Gao, & Qiu, 2015). This motivated development of a new chapter of control theory, where control and communication issues are integrated; see e.g., Antsaklis and Baillieul (2007), Mahmoud (2014), Matveev and Savkin (2009), Murray (2002), Postoyan and Nešić (2012), Postoyan et al. (2014) and Yüksel and Basar (2013).

In this area, a basic question is about the smallest communication data rate required to achieve a certain control objective for a given plant. This fundamental parameter has been studied in a variety of settings (Baillieul, 2004; de Persis, 2005; de Persis & Isidori, 2004; Liberzon & Hespanha, 2005; Matveev & Savkin, 2009; Nair, Evans, Mareels, & Moran, 2004; Nair, Fagnani, Zampieri, & Evans, 2007; Savkin, 2006) and always found to be somewhat similar to the topological entropy, which is an ubiquitous quantitative measure of randomness, chaos, uncertainty, and complexity in dynamical systems (Donarowicz, 2011; Katok, 2007). These studies gave rise to specialized control-oriented concepts of entropy. Most close to the canonical definitions (Adler, Konheim, & McAndrew, 1965; Bowen, 1971; Dinaburg, 1970) of the topological entropy is the concept accounting for uncertainties in the plant model (Savkin, 2006). Effects caused by a feedback are reckoned in the *feedback topological entropy* (Nair et al., 2004), *invariance entropy* (Colonius & Kawan, 2009), and their modifications (Colonius & Kawan, 2011; Hagihara & Nair, 2013; Kawan, 2011a): the original two concepts are shown to be essentially the same (Colonius, Kawan, & Nair, 2013).

The mentioned studies transport the topological entropy and its recent analogs from the topical areas of pure mathematics and

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physics towards everyday practice of control and communication engineers, thus enhancing the need for tractable methods of entropy evaluation. However, the last issue is highly intricate as far as nonlinearity is concerned. One of the paper's contributions is a method for constructive estimation of the entropy for nonlinear discrete-time systems.

Comprehensive closed-form expressions for entropy-related data-rate limits are well known for linear systems and cover the basic control objectives, such as stabilization and state estimation; see, e.g., the surveys in Matveev and Savkin (2009), Nair et al. (2007), Mahmoud (2014), Yüksel and Basar (2013) and Nair (2015). The "nonlinear" realm is much less inhabited by similar samples; for the most part, they are somehow close to linear systems or are one-dimensional. In Savkin (2006), topological entropy is computed in closed form for special uncertain linear systems, and this result was applied to problems of stabilization, state estimation, and optimal control. In Nair et al. (2004), closed-form computation of the local feedback topological entropy in fact deals with restriction of a smooth nonlinear system on a tight vicinity of the equilibrium, where the system is close to linear. Constructive lower and upper bounds on the invariance entropy and some its descendants are given in Kawan (2011a,b,c); conservatism of these bounds basically remains an open issue, though a closed form computation is offered for one-dimensional systems nonlinear in state and affine in control.

Intricacies in entropy evaluation are fairly well illuminated in the general theory of nonlinear dynamics. Whereas effective general approaches are known for low dimensional systems, especially for piece-wise monotone interval mappings (see, e.g., Alsedá, Llibre, & Misiurewicz, 2000; Amigó & Giménez, 2014; de Melo & van Strien, 1993; Donarowicz, 2011; Milnor & Thurston, 1988), rigorous incomputability results are obtained for more complex cases (Delvenne & Blondel, 2004; Hurd, Kari, & Culik, 1992; Koiran, 2001; Simonsen, 2006), e.g., for piece-wise affine continuous maps and $\varepsilon \approx 0$, no program can generate a rational number in a finite time that approximates the topological entropy with precision ε or better (Koiran, 2001). Exact values of topological entropy are still unknown for even such widely-studied low-dimensional chaotic systems as Hénon map, Dufing and van der Pol oscillators, Rössler system, and bouncing ball system, though various estimates and results of approximate numerical studies are available.

The first objective of this paper is to contribute to reducing conservatism of existing estimates of topological entropy via development of new tractable analytical techniques. These techniques are in the vein of the direct Lyapunov method, and the paper means to show that its potential has not been fully employed in this area up to now. This focus on the second Lyapunov method is off the main avenue of previous research, whose prevailing preferences in evaluation of the topological entropy and the likes were for Lyapunov exponents or similar instruments related to the linearized system (the first Lyapunov method) or for approaches not associated with Lyapunov. Our interest to the second Lyapunov method is partly inspired by its wide acceptance in control practice and its support by computationally efficient and theoretically well-founded algorithms based on either Linear Matrix Inequalities or the Kalman-Szegö lemma (stated in Appendix A). The proposed techniques develop some ideas from nonlinear dynamics (Boichenko & Leonov, 1995; Boichenko, Leonov, & Reitman, 2005; Douady & Osterlé, 1980; Katok, 2007; Leonov, 1991, 2008; Temam, 1997; Yomdin, 1987) and are partly based on extending our study of continuous-time plants (Pogromsky & Matveev, 2010, 2011) on systems operating in discrete time.

On the side of control theory, the paper deals with the state estimation issue for deterministic though maybe, chaotic, nonlinear systems. This is of interest in its own right and since many control problems can be typically solved if a reliable state estimate is available. The second contribution of the paper is concerned with the question: what is the minimum bit-rate of data communication from the sensor to observer which makes it possible to protect a once achieved observation accuracy from a drastic regression, or even to improve it. This question gives rise to new entropy-like characteristics of the plant, though we do not refer to them as "entropies" by retaining the name of "data rate limits".

We show that these new characteristics enhance the topological entropy; whereas the proposed techniques in fact estimate exactly them, which throws extra light on the status and scope of the techniques themselves. Their potential is demonstrated by analytically finding a closed-form expression for the exact values of these global entropy-like data rate limits for a ball vertically bouncing on a sinusoidally vibrating table, which is among the simplest physical systems that exhibit remarkably complex chaotic behaviors (Cao, Judd, & Mees, 1997; Tufillaro, Abbott, & Reilly, 1992), including existence of strange attractors (Clark, Martin, Moore, & Jesse, 1995; Mello & Tuffilaro, 1987). This system not only enjoys much attention in general study of nonlinear dynamics as a basic test example but also is of interest by its own right for various engineering applications, e.g., those concerned with jackhammers, vibro-transporters, vibratory feeders, etc. (Tufillaro et al., 1992). Our respective computation also takes the benefit of Kalman-Szegö lemma. To the best of the knowledge of the authors, the exact value of the topological entropy of the bouncing ball system still remains an open issue even for special numerical values of the parameters. Another illustration is concerned with the celebrated Hénon system (Hénon, 1976). Here the results are not so completed, partly due to the lack of analytical knowledge about invariant sets of this system. To the best of the knowledge of the authors, the paper improves the previously known closed-form estimates of the topological entropy for these two systems.

The paper is organized as follows. Section 2 offers problem setup and introduces basic concepts. Necessary and sufficient data rate bounds are reported in Sections 3 and 4, respectively. In Sections 5 and 6, they are applied to the bouncing ball and Hénon systems, respectively. In Section 7, the so obtained bounds are experimentally verified, whereas Section 8 offers brief conclusions. A complementary material and technically demanding proofs are concentrated in appendices.

The following notations are adopted in the paper: $\mathbb{Z}$ — the set of integers; $[k_1 : k_2] := \{k \in \mathbb{Z} : k_1 \leq k \leq k_2\}$; $\mathbb{Z}_+ := \{k \in \mathbb{Z} : k \geq 0\}$; $\langle \cdot ; \cdot \rangle$ and $\| \cdot \|$ — the standard inner product and Euclidean norm in $\mathbb{R}^n$, respectively; "ε-ball" — ball with the radius ε.

2. State estimation problem and basic definitions

We consider a discrete-time invariant nonlinear system

$$x(t+1) = \phi[x(t)], \quad t \in \mathbb{Z}_+, \; x(0) \in K, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, and $\phi : \mathbb{R}^n \to \mathbb{R}^n$ and $K \subset \mathbb{R}^n$ are a given continuous map and a nonempty compact set of initial states, respectively. The objective is to generate an accurate estimate $\widehat{x}(t)$ of $x(t)$ at a remote location, where direct observation of the state is impossible.

The only way to deliver information from the sensor to this location is via a discrete communication channel. At time t, it carries a discrete-valued symbol $e(t)$. So to be transmitted, continuous-valued sensor readings $x(t)$ should be first translated into such symbols. This is done by a special device, referred to as the *coder*. Its outputs are transmitted for the unit time across the channel to a *decoder* that produces an estimate $\widehat{x}(t) \in \mathbb{R}^n$ of the current state $x(t)$; see Fig. 1. Thus the *observer* is constituted by the