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Robust almost output consensus in networks of nonlinear agents with external disturbances[☆]

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ABSTRACT

This paper considers robust almost output consensus control for heterogeneous and disturbed multi-agent nonlinear systems by distributed output feedback control. Taking the effect of inherent unmodeled disturbances into account, we develop a reduced order observer based consensus protocol with a performance constraint. Our study not only encompasses internal asymptotic output consensus, but also assures certain disturbance attenuation to the closed-loop system. Substantial difficulties due to nonidentical relative degrees and directed interaction graphs are surmounted in this result. Moreover, we show that in the linear case, it comes up with a novel H_∞ almost output consensus design that improves some results recently developed in the literature.

1. Introduction

Considerable interest on consensus control has rapidly developed in the past few years for multi-agent systems from various directions; see Hong, Hu, and Gao (2006), Lin, Francis, and Maggiore (2007), Lunze (2012), Olfati-Saber and Murray (2004) and Ren and Beard (2008) for linear networks, and Ding (2013), Dong and Huang (2014), Persis and Jayawardhana (2014) and Su and Huang (2014) for nonlinear networks and references therein for up-to-date extensive studies. In the direction of output consensus (or referred to synchronization) control of heterogeneous multi-agent nonlinear systems, a number of effective internal model based methods have been developed in the literature. For example, Liu (2012) addressed the local distributed regulator design for the multi-agent nonlinear systems, Su and Huang (2014) and Isidori, Marconi, and Casadei (2014) developed some interesting semi-global consensus design, and Ding (2013) and Persis and Jayawardhana (2014) proposed global consensus design in terms of different interaction topologies.

Recently, a number of distributed control designs have been proposed for the so-called H_∞ consensus; see, e.g., Li, Liu, Fu, and Xie (2012), Oh, Moore, and Ahn (2014), Peymani, Grip, Saberi, Wang, and Fossen (2014) and Wen, Hu, Yu, and Chen (2014). These results were essentially developed for linear networks subject to external disturbances with the performance formulated as H_∞ norm of transfer function from disturbances to an average of the weighted relative outputs of the agents, or a suitably defined disagreement output. Li et al. (2012) and Oh et al. (2014) proposed some effective distributed controllers to minimizing the H_∞ norm in virtue of efficient matrix techniques. Wen et al. (2014) addressed the distributed H_∞ consensus of linear multi-agent systems with switching directed topologies. Note that all the preceding results only apply to homogeneous networks. For linear heterogeneous networks, Peymani et al. (2014) successfully developed a method to achieve arbitrarily small H_∞ gain, namely H_∞ almost synchronization.

From the developments in the aforementioned literature, we find interest in robust consensus control with disturbance attenuation for heterogeneous multi-agent nonlinear systems. To the best of our knowledge, there are rare attempts to such consensus problems. The main technical challenge would be twofold. First is that the concerned nonlinear agents are basically heterogeneous, having dynamic uncertainties and nonidentical relative degrees. This brings difficulties when we apply the regulation theory to handle the problem. In fact, the encountered augmented network does not match the block triangular form that is usually required for standard block-backstepping design.

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The second is that unlike the exosystem in the classic regulation theory of Huang (2004) and Isidori and Byrnes (1990) for modeling disturbances, some unmodeled disturbances are considered and hence the closed-loop system performance should be carefully formulated.

Contribution of the paper. A systematic approach is presented for handling robust almost output consensus control of multi-agent nonlinear systems with disturbance attenuation. For a practical starting point, we shall introduce a certain disturbance attenuation property. Moreover, it allows the nonlinear agents to be the celebrated disturbed output feedback form with nonidentical relative degrees, which removes the common relative degree condition required in the literature. Furthermore, by introducing distributed reduced order observers, our result provides a less order consensus controller than before. Last but not least, even for linear heterogeneous multi-agent systems, our result improves existing H_∞ almost output synchronization from two aspects: removing the dependence on some absolute information; reducing possible controller redundancy.

Outline. After this section, Section 2 shows the problem formulation and the motivation. Section 3 presents some solvability conditions and interpretations. Section 4 is devoted to robust almost output consensus design with disturbance attenuation. Section 5 gives an example to show the developed result. Finally, Section 6 concludes this paper.

Notation. $\|\cdot\|$ is the Euclidean norm. I is an identity matrix of an appropriate dimension. $\mathcal{K}_\infty$ is the set of continuous, strictly increasing, and unbounded functions $\alpha : [0,\infty) \to [0,\infty)$ satisfying $\alpha(0) = 0$. For any column vectors $x_1, \ldots, x_n$, $\mathrm{col}(x_1, \ldots, x_n) := [x_1^\mathsf{T}, \ldots, x_n^\mathsf{T}]^\mathsf{T}$. $A \otimes B$ means the Kronecker product of two matrices A, B.

2. Formulation and background

This paper considers a strongly coupled dynamic network of N-multiple nonlinear agents subject to affine *unmodeled* disturbance inputs described by (see e.g. Jiang, Repperger, & Hill, 2001)

$$\begin{cases} \dot{z}_i = f_i^o(z_i, y, w) + f_i^d(z_i, y, w)\varpi, \\ \dot{x}_i = A_i x_i + g_i^o(z_i, y, w) + B_i u_i + g_i^d(z_i, y, w)\varpi, \\ y_i = C_i x_i, \quad 1 \le i \le N \end{cases} \tag{1}$$

with state $(z_i, x_i) \in \mathbb{R}^{n_{z_i}} \times \mathbb{R}^{n_i}$, output $y_i \in \mathbb{R}$, and control input $u_i \in \mathbb{R}$. The constant w is an unknown parameter (or parameter uncertainties) in a fixed compact set $\mathbb{W}$. The signal $\varpi := \varpi(t) \in \mathbb{R}^m$ represents some *unmodeled* time-varying disturbances being locally essentially bounded. The matrix triplet (A_i, B_i, C_i) takes the n_i-dimensional Brunovsky normal form, i.e.,

$$A_i = \left[\begin{array}{c|c} 0 & I_{n_i-1} \\ \hline 0 & 0 \end{array}\right], \qquad B_i = [0, \cdots, 0, 1]^\mathsf{T},$$

$$C_i = [1, 0, \cdots, 0].$$

Regarding the plant (1), the integers n_i for $1 \le i \le N$ are the respective relative degrees that may be distinct in the network. The concerned agents may be coupled or connected in terms of $y = \mathrm{col}(y_1, \ldots, y_N)$. In the sense of Lunze (1992, Definition 3.1), if viewing each agent as a subsystem of (1), then the composite system can be strongly coupled through local outputs. Such systems are ubiquitous in engineering; see Lunze (1992) and Šiljak (1991) for various examples.

Besides (1), the leader as an unforced system is described by

$$\dot{v} = Sv, \qquad y_0 = q_r(v, w) \tag{2}$$

of node index 0, with state $v \in \mathbb{R}^{n_v}$ and output $y_0 \in \mathbb{R}$ as the reference to indicate desired collective behavior for output consensus. As usual, we assume that all eigenvalues of S are distinct with zero real parts and the leader (2) is invariant in a compact set $\mathbb{V}$. The functions $f_i^o(z_i, y, w)$, $f_i^d(z_i, y, w)$, $g_i^o(z_i, y, w)$, $g_i^d(z_i, y, w)$ and $q_\mu(v, w) := \mathrm{col}(q_r(v, w), \ldots, q_r(v, w))$ are sufficiently smooth in their arguments.

Relating to the composite network (1) and (2), a directed graph, called interaction digraph is denoted by a triplet $\mathcal{G} := \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ where $\mathcal{V} := \{0, 1, 2, \ldots, N\}$ is the node set, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set with no self-loop, and $\mathcal{A} = [a_{ij}]_{i,j=0,1,\ldots,N}$ is the weighted adjacency matrix. An edge of $\mathcal{G}$ is denoted by an ordered pair $(j, i) \in \mathcal{E}$ with j being a neighbor of i. In line with Godsil and Royle (2001), a directed path of $\mathcal{G}$ is an ordered sequence of distinct nodes in $\mathcal{V}$ such that any consecutive nodes in the sequence correspond to an edge of the digraph. A node j is said to be connected to another node i if there is a directed path from j to i. $\mathcal{G}$ is said to contain a directed spanning tree if there is at least one node, called the root, being connected to every other node. The matrix $\mathcal{A}$ satisfies $a_{ij} \ge 0$ and $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$ for $i, j \in \mathcal{V}$. Then an induced matrix $\mathcal{L} = [l_{ij}]_{i,j=0,1,\ldots,N}$ with $l_{ii} = \sum_{j=0}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \ne j$, is called the Laplacian of $\mathcal{G}$.

2.1. Problem formulation

To begin with, for each $i : 1 \le i \le N$, let

$$e_i = y_i - q_r(v, w), \qquad e_{mi} = \sum_{j=0}^{N} a_{ij}(y_i - y_j)$$

where a_{ij} are weights of the interaction digraph $\mathcal{G}$. In particular, $e = \mathrm{col}(e_1, \ldots, e_N)$ as the *regulated output* quantifies the consensus error and

$$e_m = He = \mathrm{col}(e_{m1}, \ldots, e_{mN}) \tag{3}$$

as *measurement output* is agent-wise available for feedback design, provided by certain output interactions between each agent and its neighbors. The regulated output e also is appointed as the *penalty variable* (cf. Doyle, Glover, Khargonekar, & Francis, 1989 and Isidori & Astolfi, 1992, Lu & Doyle, 1994) to quantify consensus performance when the signal $\varpi(t)$ does not vanish.

Problem 2.1 (*Robust Almost Output Consensus with Disturbance Attenuation*). For any prescribed leader (2), the control goal is to seek a smooth distributed control law of the form

$$\dot{\chi}_i = f_i^c(\chi_i, e_{mi}), \qquad u_i = k_i^c(\chi_i, e_{mi}) \tag{4}$$

such that, for any $(v(0), w) \in \mathbb{V} \times \mathbb{W}$ and for any initial condition $(z_i(0), x_i(0), \chi_i(0))$, the closed-loop system satisfies both the following properties.

(i) It *internally* achieves global asymptotic output consensus when $\varpi(t) \equiv 0$, i.e., its solution is bounded and the consensus error satisfies $\lim_{t\to+\infty} e(t) = 0$.

(ii) It attenuates the influence of the exogenous input $\varpi(t)$ to an arbitrarily small level, i.e., for any locally essentially bounded $\varpi(t)$, there exists a function $\gamma_\varpi \in \mathcal{K}_\infty$ such that, for any $\varepsilon > 0$ and $T > 0$, the controller (4) can be tuned to satisfy

$$\int_0^T \|e(s)\|^2 ds \le \varepsilon \left[\int_0^T \gamma_\varpi(\|\varpi(s)\|) ds + \beta(x_0^c)\right] \tag{5}$$

where $\beta(\cdot)$ is a positive definite function and x_0^c denotes the initial state of the closed-loop system.

In the preceding problem, by "almost" for robust output consensus, we mean that, in the presence of inherent unmodeled disturbances, our control allows the effect of disturbances and initial conditions on the consensus error can be made arbitrarily small in the sense of (5). One may refer to Lin, Bao, and Chen (1999) and Marino, Respondek, van der Schaft, and Tomei (1994) for an illustration of the meaning of "almost" in the setting of H_∞ control.