Automatica 70 (2016) 9–13

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Model predictive control for discrete event systems with partial synchronization☆

Xavier David-Henriet[a,b,c], Laurent Hardouin[b], Jörg Raisch[a,c], Bertrand Cottenceau[b]

[a] Technische Universität Berlin, Fachgebiet Regelungssysteme, 10587 Berlin, Germany

[b] Laboratoire Angevin de Recherche en Ingénierie des Systèmes, LARIS, ISTIA, Université d'Angers, Angers 49000, France

[c] Max-Planck-Institut für Dynamik komplexer technischer Systeme, Fachgruppe System- und Regelungstheorie, 39106 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 22 July 2014
Received in revised form
28 July 2015
Accepted 17 November 2015
Available online 13 April 2016

Keywords:
Discrete event systems
Synchronization
Model-based control
Predictive control
Transportation control
Timed Petri nets

ABSTRACT

In this paper, we consider discrete event systems divided in a main system and a secondary system such that the inner dynamics of each system is ruled by standard synchronizations and the interactions between both systems are expressed by partial synchronizations (*i.e.* event e_2 can only occur *when*, not after, event e_1 occurs) of events in the secondary system by events in the main system. The main contribution consists in adapting model predictive control, developed in the literature for $(\max, +)$-linear systems, to the considered class of systems. This problem is solved under the condition that the performance of the main system is never degraded to improve the performance of the secondary system. Then, the optimal input is selected to respect the output reference and the remaining degrees of freedom are used to ensure just-in-time behavior. The unconstrained problem is solved in linear time with respect to the length of the prediction horizon.

1. Introduction

A discrete event system is a dynamical system driven by the instantaneous occurrence of events. In the following, we focus on a particular class of time-driven (*i.e.*, the occurrence of events is only possible at clock ticks) discrete event systems, the dynamics of which is defined by synchronization rules. The standard synchronization corresponds to the following condition: occurrence k of event e_1 is *at least* τ units of time after occurrence $k-l$ of event e_2. Discrete event systems ruled only by standard synchronizations, called $(\max, +)$-linear systems, have been widely studied and admit linear state-space representations in suitable algebraic structures such as the $(\max, +)$-algebra and the $(\min, +)$-algebra (Heidergott, Olsder, & van der Woude, 2006). During the last two decades, a rich control theory has been developed for this class of systems (*e.g.*, optimal feedforward control Cohen, Gaubert, & Quadrat, 1993, model predictive control (MPC) De Schutter & van den Boom, 2001, Necoara, 2006, and model reference control Cottenceau, Hardouin, Boimond, & Ferrier, 2001). Other synchronization rules have recently been investigated such as soft synchronization (De Schutter & van den Boom, 2003) and partial synchronization (David-Henriet, Hardouin, Raisch, & Cottenceau, 2013) (*i.e.*, event e_2 can only occur *when* event e_1 occurs). Partial synchronization is useful when a system A is offering a service for a time window to another system B. For example, in a public transportation network, a passenger can access a train only *when* the train is at the train station or a car can cross an intersection only *when*, not after, the traffic light is green. In this paper, we assume that the inner dynamics of systems A and B are driven by standard synchronizations and that system A is not affected by system B. Then, the complete system is divided in a main system and a secondary system (with disjoint event sets) such that the interactions between both systems are expressed by partial synchronizations of events in the secondary system by events in the main system. We focus on the control of this kind of systems under the assumption that the performance of the main system is never degraded to improve the performance of the secondary system. This makes sense in many applications, where the main system is not only used by the secondary system, but shared by many users. For example, a train (main system) does not wait for delayed passengers (secondary system). We consider MPC, initially introduced

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Martin Guay under the direction of Editor Ian R. Petersen.

E-mail addresses: xavier.davidhenriet@gmail.com (X. David-Henriet), laurent.hardouin@univ-angers.fr (L. Hardouin), raisch@control.tu-berlin.de (J. Raisch), bertrand.cottenceau@univ-angers.fr (B. Cottenceau).

http://dx.doi.org/10.1016/j.automatica.2015.12.006
0005-1098/

for (max, +)-linear systems in De Schutter and van den Boom (2001). The unconstrained case is investigated in (min, +)-algebra, which is more suitable to deal with partial synchronizations than (max, +)-algebra (David-Henriet, Hardouin, Raisch, & Cottenceau, 2014). We prove that the unconstrained MPC optimization problem is solved at each step with a complexity linear with the length of the prediction horizon.

Necessary mathematical tools are recalled in Section 2. In Section 3, the modeling is addressed and the behavior under the earliest functioning rule is investigated. In Section 4, the main contribution of this paper, namely MPC, is introduced. Finally, the proposed control approach is applied to a supply chain in Section 5.

2. Mathematical preliminaries

The (min, +)-algebra, denoted $\overline{\mathbb{N}}_{\min}$, is defined as the set $\mathbb{N}_0 \cup \{+\infty\}$ endowed with min, denoted $\oplus$, and $+$, denoted $\otimes$. Formally, $\overline{\mathbb{N}}_{\min}$ is a dioid (Baccelli, Cohen, Olsder, & Quadrat, 1992). As in standard algebra, the product $\otimes$ is often denoted by juxtaposition (*i.e.*, ab corresponds to $a \otimes b$). The operation $\oplus$ induces an order relation $\preceq$ on $\overline{\mathbb{N}}_{\min}$ defined as

$$\forall a, b \in \overline{\mathbb{N}}_{\min}, \quad a \oplus b = b \Leftrightarrow a \preceq b.$$

Obviously, $\preceq$ corresponds to the dual of the standard order $\leq$ in $\mathbb{N}_0$ (*i.e.*, $a \succeq b \Leftrightarrow a \leq b$). In the following, only the order relation $\preceq$ is used for expressions related to $\overline{\mathbb{N}}_{\min}$. Two particular elements in $\overline{\mathbb{N}}_{\min}$ are the zero element (*i.e.*, the neutral element of $\oplus$), equal to $+\infty$ and denoted ε, and the unit element (*i.e.*, the neutral element of $\otimes$), equal to 0 and denoted e. By analogy with standard linear algebra, $\oplus$ and $\otimes$ are defined for matrices with entries in $\overline{\mathbb{N}}_{\min}$. For $A, B \in \overline{\mathbb{N}}_{\min}^{n\times m}$ and $C \in \overline{\mathbb{N}}_{\min}^{m\times p}$,

$$(A \oplus B)_{ij} = A_{ij} \oplus B_{ij} \quad \text{and} \quad (A \otimes C)_{ij} = \bigoplus_{k=1}^{m} A_{ik} C_{kj}.$$

The sum $\oplus$ and the product $\otimes$ are order-preserving. Two additional operations on $\overline{\mathbb{N}}_{\min}$ are $\wedge$, corresponding to the max in standard algebra, and $\diamond\!\!\!\backslash$, where $a \diamond\!\!\!\backslash b$ denotes the greatest solution of $a \otimes x \preceq b$. An extension of these operations to the matrix case is straightforward.

3. Input–output behavior

To capture the behavior of discrete event systems, a counter f is associated with the eponymous event f to count its occurrences. Formally, the counter f is a mapping from $\mathbb{Z}$ to $\overline{\mathbb{N}}_{\min}$, where $f(t)$ is defined as the number of occurrences of event f before or at time t. Interesting properties of counter f are

$$f(t) = e \quad \text{for } t < 0 \quad \text{and} \quad t_1 \leq t_2 \Rightarrow f(t_1) \succeq f(t_2).$$

An advantage of the counter representation is its ability to easily express standard synchronizations in $\overline{\mathbb{N}}_{\min}$. For example, the standard synchronizations "occurrence k of event e_2 is *at least* τ_1 units of time after occurrence $k - l_1$ of event $e_{1,1}$ and *at least* τ_2 units of time after occurrence $k - l_2$ of event $e_{1,2}$" correspond to the following inequality in $\overline{\mathbb{N}}_{\min}$:

$$\forall t \in \mathbb{Z}, \quad e_2(t) \succeq l_1 e_{1,1}(t - \tau_1) \oplus l_2 e_{1,2}(t - \tau_2).$$

Partial synchronization can also be expressed by a condition on counters. For example, "event e_2 can only occur *when*, not after, event e_1 occurs" is equivalent to "if event e_1 does not occur at time t, then event e_2 does not occur at time t". This corresponds to

$$\forall t \in \mathbb{Z}, \quad e_1(t) = e_1(t-1) \Rightarrow e_2(t) = e_2(t-1).$$

Next, the conditions induced by synchronizations on the dynamics of the main system and of the secondary system are formally described. For each system, the event set is partitioned into state, input, and output events as it is done for (max, +)-linear systems. The behavior of the main system must be a solution of

$$\begin{cases} x_1(t) \succeq A_{10} x_1(t) \oplus A_{11} x_1(t-1) \oplus B_1 u_1(t) \\ y_1(t) \succeq C_1 x_1(t) \end{cases}$$

where x_1, u_1, and y_1 respectively denote the vectors of counters associated with state, input, and output events in the main system. The entries of matrices A_{10}, A_{11}, B_1, and C_1 are given by the standard synchronizations in the main system. Due to partial synchronizations, the dynamics of the secondary system is slightly more complicated. The behavior of the secondary system must be a solution of

$$\begin{cases} x_2(t) \succeq A_{20} x_2(t) \oplus A_{21} x_2(t-1) \oplus B_2 u_2(t) \\ y_2(t) \succeq C_2 x_2(t) \\ \forall i, \left(\exists x_{1,j} \in \mathcal{S}_i | x_{1,j}(t) = x_{1,j}(t-1)\right) \\ \Rightarrow x_{2,i}(t) = x_{2,i}(t-1) \end{cases}$$

where x_2, u_2, and y_2 respectively denote the vectors of counters associated with state, input, and output events in the secondary system. The entries of matrices A_{20}, A_{21}, B_2, and C_2 are given by the standard synchronizations in the secondary system. The third condition expresses partial synchronizations: $\mathcal{S}_i$ denotes the set of state events in the main system synchronizing state event $x_{2,i}$ in the secondary system.

Remark 1. By analogy with (max, +)-linear systems, we assume that matrices A_{10} and A_{20} are strictly lower triangular (Baccelli et al., 1992).

Apart from the partial synchronization condition, the main and the secondary system are described by the same model structure. This allows us to treat both systems in a unified way as follows.

$$\begin{cases} x(t) \succeq A_0 x(t) \oplus A_1 x(t-1) \oplus Bu(t) \\ y(t) \succeq Cx(t) \\ \forall i, \quad \alpha_i(t) = 0 \Rightarrow x_i(t) = x_i(t-1) \end{cases} \tag{1}$$

where x, u, and y are vectors (with dimension n, p, and q) of counters associated with state, input, and output events. Mapping α_i is defined from $\mathbb{Z}$ to $\{0, 1\}$ such that $\alpha_i(t) = 1$ if, and only if, partial synchronizations allow event x_i to occur at time t. For an event x_i of the main system, $\alpha_i(t) = 1$. For an event x_i of the secondary system,

$$\alpha_i(t) = \begin{cases} 0 & \text{if } \exists x_{1,j} \in \mathcal{S}_i | x_{1,j}(t) = x_{1,j}(t-1) \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

Problem (1) only describes admissible behaviors. In the following, a standard behavior is introduced, namely the behavior under the earliest functioning rule (*i.e.*, each state or output event occurs as soon as possible). Hence, assuming that the behavior is known for times $\tau < t$, the behavior at time t is given by the least solution $(y(t), x(t))$ of (1) with the additional causality conditions $y(t) \preceq y(t-1)$ and $x(t) \preceq x(t-1)$. Checking existence and uniqueness of the behavior under the earliest functioning rule is not difficult and leads to the following input–output behavior:

$$\begin{cases} x(t) = H(x(t-1), u(t), t) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where

$$H(x(t-1), u(t), t)_i = \begin{cases} x_i(t-1) & \text{if } \alpha_i(t) = 0 \\ \displaystyle\bigoplus_{j=1}^{i-1} A_{0,ij} H(x(t-1), u(t), t)_j & \\ \quad \oplus (A_1 x(t-1) \oplus Bu(t))_i & \text{if } \alpha_i(t) = 1 \end{cases} \tag{4}$$