Automatica 70 (2016) 94–100

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Evolutionary game theoretic demand-side management and control for a class of networked smart grid[☆]

Bing Zhu[a,b,1], Xiaohua Xia[a], Zhou Wu[c,a]

[a] Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0028, South Africa
[b] School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore
[c] School of Automation, Chongqing University, Chongqing 400044, PR China

ARTICLE INFO

Article history:
Received 11 September 2015
Received in revised form
28 February 2016
Accepted 6 March 2016
Available online 15 April 2016

Keywords:
Demand-side management
Game theory
Semi-tensor product
Optimal control
Binary decision systems

ABSTRACT

In this paper, a new demand-side management problem of networked smart grid is formulated and solved based on evolutionary game theory. The objective is to minimize the overall cost of the smart grid, where individual communities can switch between grid power and local power according to strategies of their neighbors. The distinctive feature of the proposed formulation is that, a small portion of the communities are cooperative, while others pursue their own benefits. This formulation can be categorized as control networked evolutionary game, which can be solved systematically by using semi-tensor product. A new binary optimal control algorithm is applied to optimize transient performances of the networked evolutionary game.

1. Introduction

Demand-side management of energy systems becomes increasingly popular, because of its great potential in improving energy efficiency in industries. Smart grid is a typical platform where demand-side management strategies can be applied. A core issue in smart grid is that, dynamic user behaviors should be addressed in designing demand-side management strategies. Widely-used techniques for demand-side management of smart grid include game theoretic approach (Mohsenian-Rad, Wong, Jatskevich, Schober, & Leon-Garcia, 2014), multi-objective optimization (Malatji, Zhang, & Xia, 2013; Nwulu & Xia, 2015), distributed energy consumption control (Ma, Hu, & Spanos, 2014), and model predictive control (Zhang & Xia, 2011), etc.

Smart grids can be analyzed in the perspective of network systems, since there usually exist multiple interactive users consuming powers from grids. In networked smart grid systems, stability and optimization are two main issues. Stability of the networked smart grid system indicates that interactive users reach an equilibrium. Some methodologies, i.e. game theory (Mohsenian-Rad et al., 2014), can be applied to prove the existence of equilibria in networked smart grid system. Optimization of the network smart grid system implies that, in the transient process to reach the equilibrium, some indexes can be optimized. The grid provider is capable of influencing decisions of users in the network by presenting dynamic pricing strategies (Jiang, Cao, Yu, & Wang, 2014; Li, Lu, Lin, & Shen, 2013). It is possible that the smart grid provider and some of the users cooperate to affect decisions of other users, such that the common benefit can be improved.

Game theory has been widely applied to energy systems (Du, Grijalva, & Harley, 2015; Hong, Su, & Chou, 2014). In previous researches on game theoretic policy for energy systems, fundamental games are usually played between two individual users (Xiao, Mandayam, & Poor, 2015), or between the power company and users (Fadlullah, Quan, Keto, & Stojmenovic, 2014). Pay-off functions and strategies are usually defined such that existence of Nash Equilibrium (NE) can be proved. Optimization (or model predictive control Stephens, Smith, & Mahanti, 2015) can be employed to search for NE. Sometimes the fundamental game is played repeatedly, and strategies of users are updated in real-time. In this situation, it is named evolutionary game (Cheng, He, Qi, & Xu, 2015). Networked evolutionary game indicates that, the repeated game is played among networked users, and updating laws relate to topological structure of the network (Cheng, 2009).

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Kok Lay Teo under the direction of Editor Ian R. Petersen.

E-mail addresses: bing.zhu@ntu.edu.sg (B. Zhu), xxia@up.ac.za (X. Xia), wuzhsky@gmail.com (Z. Wu).

[1] Tel.: +65 67904242; fax: +65 67904242.

http://dx.doi.org/10.1016/j.automatica.2016.03.027

In some networked evolutionary games, actions of some users can be actively assigned, such that other users are induced to improve common benefit. The users with actively assigned actions can be defined as controllers; and the networked evolutionary game with controllers can be defined as control networked evolutionary game (Zhao, Li, & Cheng, 2011).

During recent years, a new semi-tensor product (Cheng, Qi, & Xue, 2007) is developed to solve the problem of networked evolutionary game. The semi-tensor product is an extension of ordinary matrix product. By using semi-tensor product, dynamics of evolutionary games can be formulated into an algebraic form (Cheng, 2009), and the existence of NE can be proved systematically (Cheng et al., 2015). For the control networked evolutionary game, control strategies can be designed to reach the NE by using semi-tensor product. Moreover, classical control methods can be introduced and extended in the framework of semi-tensor product to attain the NE of the networked evolutionary game.

In this paper, demand-side management of a class of smart grid is studied within the framework of control networked evolutionary game. The smart grid is built among interactive communities using either grid power or local generated power. It is assumed that a small portion of the communities are subsidized, thus cooperative with the grid provider. However, other communities are unsubsidized and pursuing individual benefits. We aim to design actions for the cooperative communities (controllers), such that the common benefits can be improved even if other communities are noncooperative. The main *contributions* of this paper include that: (1) the demand-side management of a smart grid is modeled into a control networked evolutionary game; (2) the networked evolutionary game is composed by fundamental games played simultaneously among several players instead of 2-player games; (3) semi-tensor product is applied to solve the demand-side management problem; and (4) a new binary optimal control is introduced to optimize the transient performance of the control networked evolutionary game.

The layout of this paper is arranged as follows. In Section 2, mathematical preliminaries are introduced. In Section 3, the demand-side management of a simple smart grid is formulated within the framework of control networked evolutionary game. In Section 4, the proposed control evolutionary game is analyzed and solved by using semi-tensor product, and a new optimal control approach is proposed to improve transient performance. In Section 5, a simulation example is presented to illustrate the proposed demand-side management approach. This paper is concluded in the final section.

2. Mathematical preliminaries

2.1. Control networked evolutionary game

Information interchange within networked system can be described by a directed graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, where $\mathscr{V} = \{\pi_1, \pi_2, \dots, \pi_n\}$ is a set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of edges that depict information flow between nodes. An edge (π_i, π_j) in $\mathscr{G}$ denotes that the information of node π_i is available to π_j, and π_i is defined as a neighbor of π_j. The index set of all neighbors of node π_j is denoted by $\mathscr{N}_j = \{i : (\pi_i, \pi_j) \in \mathscr{E}\}$. In an undirected graph, $(\pi_i, \pi_j) \in \mathscr{E} \Leftrightarrow (\pi_j, \pi_i) \in \mathscr{E}$. The adjacent matrix $\mathscr{A} \triangleq [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} = 1$ if $(\pi_j, \pi_i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. It is assumed that $a_{ii} = 0$. More details on network system can be found in Ren (2010).

Definition 1. A normal finite game $\mathscr{H}$ can be formulated by (1) the set of players: $\mathscr{V} = \{\pi_1, \pi_2, \dots, \pi_n\}$; (2) the strategy set for each player: $\mathscr{X}_i = \{x_{i1}, x_{i2}, \dots, x_{ik}\}$, where $i = 1, \dots, n$; and (3) the cost function: $c_i(x_i, x_{-i})$, where $x_i \in \mathscr{X}_i$ denotes the strategy selected by player i, and x_{-i} denotes strategies of other players excluding player i.

Definition 2. Nash equilibrium (NE), denoted by $(x_1^*, x_2^*, \dots, x_n^*)$, is a local optimal response for a normal finite game, where no individual would gain by unilaterally changing its own strategy: $c_i(x_i^*, x_{-i}^*) \le c_i(x_i, x_{-i}^*)$.

If a game can be played repeatedly with an updating law: $\Pi : x_i(t+1) = f(x_i(t), x_{-i}(t), c_i(t))$, where $t \ge 0$ denotes the discrete sampling time, then it is named evolutionary game.

In an evolutionary game played by multiple players, a typical updating law can be given by Unconditional Imitation with fixed priority (Cheng et al., 2015):

$$x_i(t+1) = x_{j^*}(t), \quad j^* = \arg\min_{j\in\mathscr{N}_i} c_j(x_j(t), x_{-j}(t)). \tag{1}$$

If j^* is non-unique, then select the minimal j^* as priority.

Definition 3. The networked evolutionary game is composed by (1) a networked graph $\mathscr{G}$; (2) a normal finite game $\mathscr{H}$ that can be played repeatedly; and (3) an updating law Π.

Remark 1. The above definition of the networked evolutionary game is slightly different from that of Cheng et al. (2015), where fundamental networked game (FNG) is required. In this paper, the normal finite game is used in Definition 3.

Definition 4. The control networked evolutionary game is composed by (1) a normal finite game $\mathscr{H}$ that is played repeatedly; (2) a networked graph $\mathscr{G}_c = (\mathscr{X} \cup \mathscr{U}, \mathscr{E})$, where $\{\mathscr{X}, \mathscr{U}\}$ is a partition of $\mathscr{V}$ ($\mathscr{X} \cup \mathscr{U} = \mathscr{V}$ and $\mathscr{X} \cap \mathscr{U} = \emptyset$), and strategies of $\mathscr{U}$ can be actively assigned; and (3) an updating law Π.

2.2. Semi-tensor product

Definition 5. The semi-tensor product of two matrix $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ can be defined by

$$A \ltimes B \triangleq (A \otimes I_{o/n})(B \otimes I_{o/p}) \in \mathbb{R}^{(mo/n)\times(qo/p)}, \tag{2}$$

where $o = \mathrm{lcm}(n, p)$ denotes the least common multiple of n and p; and $\otimes$ denotes the Kronecker product.

Definition 6. The fundamental vector $\delta_n^i \in \mathscr{D}_n$ is defined as the ith column of the identity matrix $I_{n\times n}$. It can be further defined that $\delta_n[i, j, \dots, k] \triangleq [\delta_n^i, \delta_n^j, \dots, \delta_n^k]$.

Theorem 1 (Cheng et al., 2015). *With equivalence $i \sim \delta_n^i$, $i = 1, 2, \dots, n$, a logic function $f : \mathscr{D}_n^k \to \mathscr{D}_n$ can be rewritten by $f(x_1, x_2, \dots, x_k) = M_f \ltimes_{i=1}^k x_i$, where M_f is the structure matrix of logic function f.*

Theorem 2 (Cheng et al., 2015). *For a logic dynamic system $x_i(t+1) = f_i(x_i(t), x_{-i}(t)) = M_{fi} \ltimes_{i=1}^n x_i$, $i = 1, \dots, n$, it can be rewritten in the form of*

$$x(t+1) = M_f x(t), \tag{3}$$

where $x(t) \triangleq \ltimes_{i=1}^n x_i$, and $M_f \triangleq M_{f1} * M_{f2} * \cdots * M_{fn}$. Here, $*$ denotes the Khatri–Rao product: $M * N \triangleq [\mathrm{col}_1(M) \ltimes \mathrm{col}_1(N), \dots, \mathrm{col}_s(M) \ltimes \mathrm{col}_s(N)]$, where $M \in \mathbb{R}^{p\times s}$ and $N \in \mathbb{R}^{q\times s}$; and $\mathrm{col}_i(M)$ denotes the ith column of matrix M.

Theorem 3 (Cheng et al., 2015). *For a logic dynamic system given by (3), δ_n^i is its fixed point, if and only if the diagonal element m_{ii} of M_f equals 1.*