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Brief paper

Pulse width modulation for multi-agent systems☆

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ABSTRACT

This paper studies the consensus problem for multi-agent systems. A distributed consensus algorithm is developed by constructing homogeneous pulse width modulators for agents in the network. In particular, a certain percentage of the sampling period named duty cycle is modulated according to some state difference with respect to the neighbors at each sampling instant. During each duty cycle, the amplitude of the pulse is fixed. The proposed pulse width modulation scheme enables all agents to sample asynchronously with arbitrarily large sampling periods. It provides an alternative digital implementation strategy for multi-agent systems. We show that consensus is achieved asymptotically under the proposed scheme. The results are compared with the self-triggered ternary controller.

1. Introduction

Pulse width modulation (PWM) is one of the most frequently used ways to perform analog-to-digital conversion with applications in diverse areas including signal processing, control, communication, and power electronics (Skoog & Blankenship, 1970). Ease of implementation makes the utilization of PWM an attractive alternative in many control systems (Wang, Meng, & Chen, 2014). PWM uses rectangular pulse waves with fixed amplitude while the pulse width is adjusted during each period. All pulses have the same amplitude during the duty cycle of the period, but the sign is determined at the beginning of each period according to the control objective. PWM shares the same philosophy as event triggered control, which has been shown to be efficient in utilization of communication and computational resources (Meng & Chen, 2012; Ramesh, Sandberg, & Johansson, 2013; Sánchez, Guarnes, & Dormido, 2009). Both PWM and event triggered control can be regarded as state-dependent switching control laws. In the PWM scheme, the time when the control signal switches from "on" to "off" depends on the sampled state at the beginning of each cycle.

A multi-agent system is a system composed of multiple interacting intelligent agents. Typical multi-agent systems include multiple spacecraft, fleets of autonomous rovers, and formations of unmanned aerial vehicles. The research interest in consensus problems for multi-agent systems is evident with recent monographs (Mesbahi & Egerstedt, 2010; Ren & Beard, 2008) and papers (Liu, Li, Xie, Fu, & Zhang, 2013; Meng, Ren, & You, 2010; Qin, Zheng, & Gao, 2011; Xiao & Wang, 2008). Early control algorithms for consensus problems are based on continuous information exchange with the assumption that the communication bandwidth is sufficiently large. However, the communication bandwidth is often limited in reality. Therefore, a digital implementation of multi-agent systems is much desired.

In this paper, we explore the consensus problem for multi-agent systems with PWM. After obtaining neighbors' information, each agent converts the information into the width of a rectangular pulse wave with unit amplitude. Then the pulse wave is applied to the local agent as an input signal. In contrast to existing results on digital control for multi-agent systems, the main contributions lie in the following four aspects: complete distribution, asynchronous sampling, arbitrarily large sampling period, and saturation free.

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Firstly, the proposed algorithm is completely distributed in the sense that we require only neighbors' information instead of global topology information, such as the largest or the smallest positive eigenvalues of the associated graph Laplacian matrix. This supports a plug-and-play implementation easily handling agents added to or removed from the network. Secondly, we show that asynchronous sampling is possible for the proposed PWM scheme. Thirdly, we demonstrate that the sampling period can be arbitrarily large for asymptotic consensus. Lastly, the PWM algorithm with a fixed amplitude is advantageous to deal with actuator saturation.

Notation. Let $\mathbb{Z}^+$ be the set of non-negative integers, that is, $\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$. The sign function is defined as $\operatorname{sgn}(z) = 1$ if $z > 0$, $\operatorname{sgn}(z) = 0$ if $z = 0$, and $\operatorname{sgn}(z) = -1$ if $z < 0$. For a given real number c, $\lceil c \rceil$ denotes the smallest integer larger than or equal to c.

2. Problem formulation

2.1. Algebraic graph theory

Digraphs $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ are frequently used to model information exchange among agents, where the vertex set $\mathcal{V} = \{1, \ldots, N\}$ represents agents in a network, and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ characterizes the connectivity between agents. The set of neighbors of node i is denoted $\mathcal{N}_i := \{j : (j, i) \in \mathcal{E}\}$ and $|\mathcal{N}_i|$ is the neighborhood cardinality. A directed path is a non-empty subgraph $\mathcal{G}' = (\mathcal{V}', \mathcal{E}')$ of $\mathcal{G}$ of the form $\mathcal{V}' = \{i_0, i_1, \ldots, i_k\}$, $\mathcal{E}' = \{(i_0, i_1), (i_1, i_2), \ldots, (i_{k-1}, i_k)\}$ where the i_j, $j = 0, 1, \ldots, k$ are all distinct. A (non-empty) directed graph is said to have a directed spanning tree if there exists at least one node having a directed path to all other nodes.

2.2. System model

The dynamics of each agent obeys a single integrator model

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{1}$$

where $x_i(t)$ is a scalar and $u_i(t)$ denotes the control input for each agent. A distributed PWM algorithm is considered here in the sense that each agent receives information only from neighbors. Also note that each agent has access to only the relative state differences from neighbors with respect to its own state. The information from neighbors will be modulated and then applied as a control input. PWM strategy guarantees a strictly positive lower bound of inter-sample periods for each agent and thus rules out Zeno behavior (Johansson, Egerstedt, Lygeros, & Sastry, 1999).

2.3. Distributed PWM

Let us first define some terminologies. *Sampling instants* $\{kh_i, k \in \mathbb{Z}^+\}$ are the instants when agent i measures the relative differences with respect to all its neighbors $j \in \mathcal{N}_i$ periodically with a fixed *sampling period* h_i. The PWM control scheme can be described as follows. On each period the input u_i for agent i is switched exactly once from either 1 or -1 to 0. The length of the duration of the kth sampling period on which the input holds the fixed value 1 or -1 is known as the *duty cycle* α_i^k and the *duty rate* is denoted α_i^k/h_i. The duty cycle depends on the state, which will be shown later. The PWM control scheme originates from the control of switching power converters, where usually it is reasonable to assume that the switches can be "on" and "off" at any ratio $\alpha_i^k/h_i \in [0, 1)$.

Let us define an indicator function $s_i(t)$ for agent i to describe "on" and "off" times over a sampling period. When $\alpha_i^k = 0$, $s_i(t) = 0$ for $t \in [kh_i, kh_i + h_i)$; when $\alpha_i^k \neq 0$, $s_i(t) = 1$ if $t \in [kh_i, kh_i + \alpha_i^k)$, and $s_i(t) = 0$ if $t \in [kh_i + \alpha_i^k, kh_i + h_i)$. The length of the duty cycle for agent i at sampling instant kh_i is defined as $\alpha_i^k = 0$ if $\mathcal{N}_i = \emptyset$ or $z_i(kh_i) = 0$, and

$$\alpha_i^k = \min\left\{\frac{|z_i(kh_i)|}{2|\mathcal{N}_i|},\ h_i\right\}, \tag{2}$$

otherwise, where

$$z_i(kh_i) = \sum_{j \in \mathcal{N}_i} \left(x_i(kh_i) - x_j(kh_i)\right).$$

Intuitively, each agent measures the sum of the disagreement with respect to its neighbors, and sets the length of its duty cycle proportional to the discrepancy. We define the piecewise constant signal

$$\hat{z}_i(t) = z_i(kh_i), \quad \text{for } t \in [kh_i, kh_i + h_i),$$

and let the control input for agent i be given by

$$u_i(t) = -s_i(t)\operatorname{sgn}\hat{z}_i(t). \tag{3}$$

The solution notion for the differential equation (1) with (3) can be defined using the notion of sample-and-hold solution (Clarke, Ledyaev, Sontag, & Subbotin, 1997).

Remark 1. The sample pattern here is different from the traditional sample-and-hold case (Xie, Liu, Wang, & Jia, 2009). Here each agent samples the neighbors' information periodically in an asynchronous way. Note also that the sampling periods for distinct agents are different. The PWM algorithm allows a distributed implementation without using any a priori information about the global topology. Our PWM scheme shares the philosophy of event triggered control since the length of the pulse depends on the sampled state information.

Remark 2. The PWM algorithm is similar to the finite time consensus algorithm in Cortés (2006) and the ternary controller in De Persis and Frasca (2013), as it uses $\{-1, 0, 1\}$ as the control input set. The PWM algorithm is different from those algorithms in information acquisition and utilization. The finite time consensus algorithm in Cortés (2006) requests neighbors' state and updates the controller continuously, while the ternary controller in De Persis and Frasca (2013) uses self-triggered communication and piecewise constant control between two consecutive sampling instants. The PWM scheme obtains the information periodically, and the control signal is switched once during each period.

The objective of this paper is to propose a PWM algorithm such that global asymptotic consensus is achieved for the multi-agent system (1).

Definition 3. The multi-agent system (1) with a given PWM algorithm u_i, for all $i \in \mathcal{V}$, achieves global asymptotic consensus if for all $x_i(0) \in \mathbb{R}$ and all $i \in \mathcal{V}$, it holds that $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$, for all $i, j \in \mathcal{V}$.

3. PWM over directed graphs

Without loss of generality, we relabel $\mathcal{V} = \{1, 2, \ldots, N\}$ such that $0 < h_1 \le h_2 \le \cdots \le h_N$. Define $\Phi(x) = \max_{i\in\mathcal{V}} x_i$, $\Psi(x) = \min_{i\in\mathcal{V}} x_i$, and $V(x) = \Phi(x) - \Psi(x)$, where $x = [x_1, x_2, \ldots, x_N]^{\mathrm{T}}$. In addition, denote $\Phi^* = \Phi(x(0))$, $\Psi^* = \Psi(x(0))$. Before giving the main result, we first present two supporting lemmas. The following lemma shows that the states of all agents of the system (1) with the control law (3) remain bounded for all $t \ge 0$, where the proof is given in Appendix A.