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Brief paper

Synchronizing nonlinear complex networks via switching disconnected topology[☆]

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ABSTRACT

The current theoretical study on the synchronization mechanism of complex networks mainly focuses on fixed connected topology or switching balanced topology, which is heavily based on constructing common smooth Lyapunov functions. However, for complex networks with nonlinear node dynamics, it is still unknown whether such kind of networks can synchronize under the condition of switching, directed, and disconnected topology. By employing the ideas of sequential connectivity and joint connectivity, this paper finds that complex networks with one-sided Lipschitz node dynamics can realize synchronization even if the network topology is not connected at any time instant. By iteratively estimating the maximal distance between different nodes, this paper gives several sufficient conditions on synchronization of nonlinear complex networks under switching disconnected topology. Finally, simulation examples validate the main results.

1. Introduction

Complex networks are a typical type of complex systems, representing the complex interactions among different components. Multitudes of social, biological and engineering systems can be described and analyzed by complex networks (Albert & Barabási, 2002). Historically, the investigation of complex networks can be traced back to the discovery of the fundamental scale-free and small-world networks (Strogatz, 2001; Watts & Strogatz, 1998). Based on these innovative models, many interesting results have been reported in the field of complex networks, towards to the understanding of the origin of complexity.

Synchronization is one of the typical collective behaviors of complex networks (Arenas, Diaz-Guilera, Kurth, Moreno, & Zhou, 2008), which means the states of a system reach some identical value or trajectory asymptotically. Synchronization also has some important engineering applications, such as clock synchronization of sensor networks and distributed power generation (Blaabjerg, Teodorescu, Liserre, & Timbus, 2006). During the past decade, a great number of interesting results have been reported in the field of network synchronization from different perspectives (Lü & Chen, 2005). It should be pointed out that most of the existing results only concern the dynamical behavior of complex networks with fixed topology, which can be generally attributed to the master equation method (Li, Duan, Chen, & Huang, 2010; Pecora & Carroll, 1998) or Lyapunov method (Yu, Chen, & Lü, 2009). Nevertheless, due to the complex environment in which the networks are involved, the network topology itself may also be dynamic, called dynamic or switching topology. However, there are few reported results focusing on synchronization of nonlinear complex networks with switching topology, especially disconnected topology.

The reliability of communication links in complex networks plays a critical role for network synchronization. Though existing results demonstrate the synchronizability of complex networks under the condition of fixed topology (Li et al., 2010), it remains an interesting issue to ask whether complex networks can realize synchronization under imperfect communication. Recently, in the field of multi-agent systems (MAS), some interesting results

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demonstrate that MAS can reach synchronization under the condition of fast switching topology (Kim, Shim, Back, & Seo, 2013), jointly connected and sequentially connected topology (Angeli & Bliman, 2009; Chen, Lü, Yu, & Hill, 2013; Chen, Yu, Li, & Feng, 2013; Hong, Gao, Cheng, & Hu, 2007; Jadbabaie, Lin, & Morse, 2003; Ren & Beard, 2005). Here, joint (or sequential) connectivity means the topology is not necessarily connected at any time instant but connected over a time interval (Cao, Morse, & Anderson, 2008). Inspired by these findings, it is natural to ask whether complex networks can reach synchronization under the condition of sequential or joint connectivity.

As far as we know, there does not exist a result which discusses the synchronization of nonlinear complex networks under the condition of jointly connected topology. Furthermore, even for the simplest case of networked linear system with switching topology, the widely used quadratic or smooth Lyapunov function does not exist (Brayton & Tong, 1979; Olshevsky & Tsitsiklis, 2008). Considering the nonlinear dynamics involved in the model of complex networks, solving such a problem becomes even more difficult. Therefore, it is necessary to develop novel techniques to handle the synchronization problem in case of switching topology. The major contribution of this paper is: it employs the ideas of sequential and joint connectivity, and proves that complex networks with one-sided Lipschitz node dynamics can realize synchronization under the condition of switching, directed, and disconnected topology.

This paper is organized as follows: Section 2 gives some preliminaries on graph theory; Section 3 presents the main results; Section 4 illustrates the details of technical proof; Section 5 gives a simulation example to verify the main results; Section 6 concludes this paper.

2. Preliminaries

A graph $\mathcal{G} = \{V, E\}$ is composed of two sets: V is the set of nodes and $E \subseteq V \times V$ is the set of edges. Given two graphs $\mathcal{G}_1 = \{V, E_1\}$ and $\mathcal{G}_2 = \{V, E_2\}$, the union of these two graphs is defined by $\mathcal{G}_1 \cup \mathcal{G}_2 = \{V, E_1 \cup E_2\}$. A graph $\mathcal{G} = \{V, E\}$ is called strongly connected if there exists a path from any node $i \in V$ to $j \in V$ $(i \neq j)$. A graph $\mathcal{G} = \{V, E\}$ is called connected if for any pair of vertices i, j $(i \neq j)$, there exists a path from i to j or from j to i. If $\mathcal{G}$ is connected, then $\mathcal{G}$ contains a spanning tree. In this paper, we focus on complex networks with topologies described by directed graphs.

Given any nonnegative matrix $A = (a_{ij})_{i,j=1}^N \in \mathbb{R}^{N\times N}$, the graph corresponding to A is given by $\mathcal{G}(A) = \{V, E\}$, that is $(j, i) \in E$ if and only if $a_{ij} > 0$.

Given a graph $\mathcal{G} = \{V, E\}$, for any $\mathcal{V} \subseteq V$, define

$$\partial(\mathcal{G}, \mathcal{V}) = \{j : i \in \mathcal{V},\ (i, j) \in E\}.$$

Intuitively speaking, $\partial(\mathcal{G}, \mathcal{V})$ is the set of nodes in which each one has a link pointed from a corresponding node in $\mathcal{V}$.

A sequence of graphs $\{\mathcal{G}_i\}_{i=1}^m$ is jointly connected if $\bigcup_{i=1}^m \mathcal{G}_i$ contains a spanning tree (Jadbabaie et al., 2003).

A sequence of graphs $\{\mathcal{G}_i\}_{i=1}^m$ is sequentially connected (Angeli & Bliman, 2009) if there exists $\mathcal{V}_k \subseteq V$ $(1 \le k \le m+1)$ satisfying[2]

$$\mathcal{V}_{k+1} \subseteq \partial(\mathcal{G}_k, \mathcal{S}_k), \quad \mathcal{S}_k = \bigcup_{s=1}^k \mathcal{V}_s,$$

where $\mathcal{S}_1 = \mathcal{V}_1$ is a singleton and $\mathcal{S}_{m+1} = V$.

[2] The definition of sequential connectivity for continuous-time networks (1) is different from that of discrete-time networks in that: there is no self-loop required in the continuous-time case.

The intuitive meaning of sequential connectivity can be explained as follows. When $\{\mathcal{G}_i\}_{i=1}^m$ is sequentially connected, there exists a node, called i^* (i.e. $\mathcal{S}_1 = \{i^*\}$), sends information to its neighbors $\mathcal{V}_2$ at time t_1, then those who have received the information, called $\mathcal{S}_2$, send the information to their neighbors $\mathcal{V}_3$ at time t_2, repeat the above process and finally all the nodes can get this information at time t_{m+1}.

3. Model, assumptions and main results

Let $V = \{1, 2, \ldots, N\}$ be the set of nodes and consider the following complex network with nonlinear node dynamics (Yu et al., 2009)

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^N a_{ij}(t)(x_j(t) - x_i(t)), \quad i \in V. \tag{1}$$

Here, $x_i(t) \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear vector function, $c \in \mathbb{R}$ is the coupling coefficient, and $a_{ij}(t) \in \mathbb{R}$ are piecewise functions satisfying $a_{ij}(t) \equiv 0$ or $a_{ij}(t) \equiv 1$ in each interval $[t_k, t_{k+1})$,[3] where $t_k = (k-1)h$ and h is the sampling step.[4] Let $a_{ii}(t) = 0$ for any $t \ge 0$ and $i \in V$. Denote that

$$A_k = (a_{ij}(t))_{i,j=1}^N$$

in $[t_k, t_{k+1})$. Synchronization of complex networks (1) implies that

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$$

for any $i, j \in V$.

The following assumptions are needed throughout the paper:

Assumption 1. The nonlinear function f is one-sided Lipschitz, which means there exists $L \in \mathbb{R}$ such that for any $x, y \in \mathbb{R}^n$, the following inequality holds:

$$(x-y)^T(f(x) - f(y)) \le L\|x-y\|^2.$$

Assumption 2. There exists some integer $T > 0$ such that the sequence of graphs $\{\mathcal{G}_k\}_{k=(r-1)T+1}^{rT}$ is sequentially connected for any $r \ge 1$, $ch \le 1$, and

$$e^{LTh}\left(1 - (che^{-2(N-1)ch})^T\right) < 1. \tag{2}$$

Assumption 1 stresses the requirement for the nonlinear function $f(\cdot)$, Assumption 2 establishes the relationship between coupling coefficient c and sampling interval h under the condition of sequentially connected topology, respectively.

According to Abbaszadeh and Marquez (2010), any Lipschitz function is one-sided Lipschitz, but the converse is not true. This means that for some nonlinear function with big Lipschitz constant, the corresponding one-sided Lipschitz constant L may be very small or even negative (see, the example given in Section 5). In the case that $L \le 0$, it is easy to verify the condition 2 is naturally satisfied, leading to the unconservatism of the given assumptions.

The main result of this paper can be illustrated as follows.

Theorem 3. *If Assumptions 1 and 2 hold, then complex network (1) reaches synchronization.*

[3] We make this requirement only to simplify the technical proof, $a_{ij}(t)$ can be generalized to the time-varying case by using the same method of this paper.

[4] By using the same method in Section 4, the sampling interval h can be generalized to non-evenly distributed h_k, we omitted the detailed deduction due to restriction of space.