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Brief paper

Stability of predictor-based feedback for nonlinear systems with distributed input delay ☆

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ABSTRACT

We consider Ponomarev's recent predictor-based control design for nonlinear systems with distributed input delays and remove certain restrictions to the class of systems by performing the stability analysis differently. We consider nonlinear systems that are not necessarily affine in the control input and whose vector field does not necessarily satisfy a linear growth condition. Employing a nominal feedback law, not necessarily satisfying a linear growth restriction, which globally asymptotically, and not necessarily exponentially, stabilizes a nominal transformed system, we prove global asymptotic stability of the original closed-loop system, under the predictor-based version of the nominal feedback law, utilizing estimates on solutions. We then identify a class of systems that includes systems transformable to a completely delay-free equivalent for which global asymptotic stability is shown employing similar tools. For these two classes of systems, we also provide an alternative stability proof via the construction of a novel Lyapunov functional. Although in order to help the reader to better digest the details of the introduced analysis methodology we focus on nonlinear systems without distributed delay terms, we demonstrate how the developed approach can be extended to the case of systems with distributed delay terms as well.

1. Introduction

1.1. Background and motivation

In Ponomarev (in press) the following class of systems is considered

$$\dot{X}(t) = f(X(t)) + B_1(X(t))U(t-D) + B_0(X(t))U(t) + \int_{-D}^{0} B_{\text{int}}(\theta, X(t))U(t+\theta)d\theta, \quad (1)$$

where $X \in \mathbb{R}^n$ is state, $U \in \mathbb{R}$ is control input, $D > 0$ is a delay, $t \in \mathbb{R}$ is time, $f : \mathbb{R}^n \to \mathbb{R}^n$ is vector field, and $B_0, B_1 : \mathbb{R}^n \to \mathbb{R}^n$ and $B_{\text{int}} : [-D, 0] \times \mathbb{R}^n \to \mathbb{R}^n$ are input vector fields. A predictor-based control law is designed in Ponomarev (in press) for the stabilization of (1).

In this article, we consider the following system

$$\dot{X}(t) = f(X(t), U(t-D), U(t)), \quad (2)$$

under a predictor-based control law that is constructed employing the design tools introduced in Ponomarev (in press).

Numerous recent results on the predictor-based stabilization of nonlinear systems controlled only through a single input channel with delay are reported, including systems with constant (Krstic, 2009, 2010; Mazenc & Malisoff, 2014), state-dependent (Bekiaris-Liberis & Krstic, 2013a,b,c), input-dependent (Bresch-Pietri, Chauvin, & Petit, 2014), and unknown (Bresch-Pietri & Krstic, 2014) delay, systems stabilized under sampling (Karafyllis & Krstic, 2012), positive systems (Mazenc & Niculescu, 2011), as well as the introduction of approximation and implementation schemes (Karafyllis, 2011; Karafyllis & Krstic, 2013, 2014). Despite the several recent developments, the problems of stabilization and of stability analysis of nonlinear systems of the form (1) and (2) are rarely investigated (Mazenc, Niculescu, & Bekaik, 2013; Ponomarev, in press) (see also Marquez-Martinez & Moog, 2004; Xia, Marquez-Martinez, Zagalak, & Moog, 2002 that adopt an algebraic approach), although both predictor-based design techniques, including classical reduction approaches (Artstein, 1982; Manitius &

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Olbrot, 1979; Mondie & Michiels, 2003), optimal (Ariola & Pironti, 2008; Shuai, Lihua, & Huanshui, 2008) and robust (Chen & Zheng, 2002; Yue, 2004) control methods, and nested prediction-based control laws (Zhou, 2014), as well as analysis tools (Bekiaris-Liberis & Krstic, 2011; Fridman, 2014; Li, Zhou, & Lam, 2014; Mazenc, Niculescu, & Krstic, 2012; Ponomarev, 2016) exist for the linear case.

Besides the theoretical significance of studying systems of the form (1) and (2), which lies in the fact that the classic linear predictor-based control design approach is extended to the nonlinear case, systems of the form (1) and (2) appear in various applications such as networked control systems (Goebel, Munz, & Allgower, 2010; Roesch, Roth, & Niculescu, 2005), population dynamics (Artstein, 1982), and combustion control (Xie, Fridman, & Shaked, 2001; Zheng & Frank, 2002), among several other applications (Niculescu, 2001; Richard, 2003).

1.2. Contribution

For system (2) we design a predictor-based control law following the design procedure developed in Ponomarev (in press). Specifically, we first define the transformation Z of the state X defined as

$$p(x,t) = X(t) + \int_0^x f\left(p(y,t), u(y,t), 0\right) dy, \quad x \in [0, D] \tag{3}$$

$$Z(t) = p(D,t), \tag{4}$$

where we use the following, transport PDE representation of the actuator state $U(\theta), \theta \in [t-D, t]$,

$$u_t(x,t) = u_x(x,t), \quad x \in [0, D] \tag{5}$$

$$u(D,t) = U(t), \tag{6}$$

which transforms system (2) to a new system of the form[1]

$$\dot{Z}(t) = F\left(Z(t), U_t, U(t)\right), \tag{7}$$

where the function U_t is defined by $U_t(s) = U(t+s)$, for all $s \in [-D, 0]$. The control law that stabilizes system (7)[2] is given for all $t \geq 0$ by

$$U(t) = \kappa\left(Z(t), U_t\right). \tag{8}$$

Although the predictor-based design (8), (4), (3) is derived by employing the design methodology developed in Ponomarev (in press), in this article we introduce novel stability analysis tools, which, in comparison with Ponomarev (in press), allows one to remove the plant and controller growth restrictions, as well as the requirements that the control be affine and that the nominal controller achieves exponential stability of the transformed system. Specifically, we prove global asymptotic stability for systems that are not necessarily affine in the control, without necessarily imposing a linear growth condition either on the vector field or the nominal controller and without assuming that the nominal control law achieves exponential stability. Our stability analysis is based on estimates on closed-loop solutions.

We also identify a class of systems that includes systems transformable to a completely delay-free equivalent and which we categorize into two different types of systems. For this class of systems we also construct a novel Lyapunov functional with the aid of which we prove global asymptotic stability of the closed-loop system, thus providing an alternative stability proof.

[1] For the sake of clarity of presentation the exact form of F is given in Section 2.

[2] The specific properties of the closed-loop system and κ are specified in Section 2.

Although in order to help the reader to better understand the conceptual ideas of our methodology we concentrate on systems of the form (2), i.e., without distributed delay terms, the same tools can be applied to systems with distributed delay terms of the form

$$\dot{X}(t) = f\Bigg(X(t), U(t-D), \int_{t-D}^t b_1(\theta - t)U(\theta)d\theta, \ldots, \int_{t-D}^t b_m(\theta - t)U(\theta)d\theta, U(t)\Bigg). \tag{9}$$

1.3. Organization

In Section 2 we prove global asymptotic stability under predictor-based feedback for general nonlinear systems. In Section 3 we identify a class of systems that includes systems transformable to a delay-free equivalent. For this class of systems we construct a Lyapunov functional with the aid of which we prove global asymptotic stability under predictor-based feedback in Section 4. We illustrate the fact that the developed approach can be applied to systems with distributed delay terms in Section 5.

Notation: We use the common definition of class $\mathcal{K}$, $\mathcal{K}_\infty$, and $\mathcal{KL}$ functions from Khalil (2002). For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. For a function $u : [0, D] \times \mathbb{R}_+ \to \mathbb{R}$ we denote by $\|u(t)\|_\infty$ its spatial supremum norm, i.e., $\|u(t)\|_\infty = \sup_{x\in[0,D]} |u(x,t)|$. For any $c > 0$, we denote the spatially weighted supremum norm of u by $\|u(t)\|_{c,\infty} = \sup_{x\in[0,D]} |e^{cx}u(x,t)|$. For a vector valued function $p : [0, D] \times \mathbb{R}_+ \to \mathbb{R}^n$ we use a spatial supremum norm $\|p(t)\|_\infty = \sup_{x\in[0,D]} \sqrt{p_1(x,t)^2 + \cdots p_n(x,t)^2}$. For a function $U : [-D, \infty) \to \mathbb{R}, \forall t \geq 0$, the function U_t is defined by $U_t(s) = U(t+s), \forall s \in [-D, 0]$. We denote by $C^j(A; E)$ the space of functions that take values in E and have continuous derivatives of order j on A.

Solutions: We assume that the initial condition $U_0 \in C([-D, 0]; \mathbb{R})$ is compatible with the feedback law (8), i.e., it holds that $U_0(0) = \kappa(Z(0), U_0)$, such that under the assumptions that $\kappa : \mathbb{R}^n \times C([-D, 0]; \mathbb{R}) \to \mathbb{R}$ is locally Lipschitz and that $f : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^n$ is twice continuously differentiable (Assumption 1 in Section 2), which allows one to conclude that $F : \mathbb{R}^n \times C([-D, 0]; \mathbb{R}) \times \mathbb{R} \to \mathbb{R}^n$ is locally Lipschitz,[3] there exists a unique solution $Z(t) \in C^1([0, \infty), \mathbb{R}^n)$ and $U(t) \in C([0, \infty), \mathbb{R})$ (see Hale & Verduyn Lunel, 1993; Karafyllis, Pepe, & Jiang, 2009; Mazenc et al., 2012; Pepe, 2007; Pepe, Karafyllis, & Jiang, 2008),[4] which in turn implies from (2) that there exists a unique solution $X(t) \in C^1([0, \infty), \mathbb{R}^n)$.

2. Stability analysis for general systems

Assumption 1. The vector field $f : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^n$ is twice continuously differentiable with $f(0,0,0) = 0$ and satisfies

$$f(X, \omega, \Omega) - f(X, \omega, 0) = g(X, \Omega) \tag{10}$$

for all $(X, \omega, \Omega)^T \in \mathbb{R}^{n+2}$ and some $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$.

[3] The Lipschitzness of F (Lemma 1 in Section 2) follows by the regularity of f and the Lipschitzness of the solutions to $p_x(x) = f(p(x), u(x), 0)$, $p(D) = Z$ with respect to $Z \in \mathbb{R}^n$ and $u \in C([0, D]; \mathbb{R})$, as well as to $r_x(x) = \frac{\partial f(p(x), u(x), 0)}{\partial p} r(x)$, $r(0) = g$ with respect to $g \in \mathbb{R}^n$, $p \in C([0, D]; \mathbb{R}^n)$ and $u \in C([0, D]; \mathbb{R})$ (see, e.g., Hale & Verduyn Lunel, 1993; Khalil, 2002).

[4] The fact that $Z(t)$ and $U(t)$ are defined on $[0, \infty)$ follows from the stability properties of system (7), (8), which are established employing Assumption 3 in Section 2.