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Brief paper

Decentralized H-infinity control of complex systems with delayed feedback☆

Lubomír Bakule, Branislav Rehák, Martin Papík

Institute of Information Theory and Automation, Czech Academy of Sciences, 182 08 Prague 8, Czech Republic

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ABSTRACT

The paper studies the problem of decentralized H_∞ fault tolerant state feedback control design for a class of continuous-time complex systems composed of identical subsystems and symmetric interconnections. We consider a time-varying interval-bounded delay in the feedback of each channel. Single delay as well as multiple delay cases is considered. By exploiting a particular structure of the systems, sufficient conditions are derived for the gain matrix selection. The controller design is performed using a reduced-order system under linear matrix inequality approach constraints. The asymptotic stability with disturbance attenuation γ of the overall multiple delay closed-loop system is guaranteed when synthesizing the gain matrix into the decentralized controller. Moreover, sufficient conditions for the H_∞ bound tolerance under local control channel failures of the overall closed-loop system are derived. The tolerance can be easily tested on several low-order systems. A numerical example illustrates the effectiveness of the proposed method.

1. Introduction

The paper is focused on a class of continuous-time dynamic systems composed of the interconnection of identical subsystems with identical couplings. Such systems are known as symmetrically interconnected systems. They appear in very different real world systems as presented for instance in Bakule (2005), Bakule (2007) and Bakule and Rodellar (1996). This paper shows that such a structure of subsystems and interconnections enables a special analysis and control design procedure. The main feature of this method is the setup of systems with reduced dimension, but keeping the same dynamic properties. A more comprehensive survey of theoretical and applied results can be found for instance in Bakule (2014) with the references therein.

1.1. Prior work

The paper is mainly inspired by the previous works on symmetric composite systems with delayed feedback presented in Bakule, de a Sen, Papík, and Rehák (2013a) and Bakule, de la Sen, Papík, and Rehák (2013b), as well as the results on H_∞ fault tolerance for this class of systems in Huang, Lam, Yang, and Zhang (1999) and Lam and Huang (2007) or the LQ reliable control studied by Huang, Lam, and Yang (2001). It is well known that the synthesis can be simplified through an appropriate transformation to a set of systems of reduced dimensions.

The first contribution of the paper is a procedure for the gain selection of the decentralized H_∞ delayed feedback which guarantees the asymptotic stability with the disturbance attenuation level γ of the overall multiple delay closed-loop system. A convex optimization approach is used for the state feedback matrix gain selection by Peng and Tian (2007) originally extended by the authors into the robust control setting by using Cao, Sun, and Lam (1998). It is shown how to synthesize this gain matrix into the overall system.

The second contribution is sufficient conditions for the asymptotic stability with the disturbance attenuation level γ of the overall multiple delay closed-loop systems if several local feedback controllers fail. The problem is to find an integer which corresponds with the smallest number of failures that make the global closed-loop system unstable or will cause the violation of the disturbance attenuation level γ. Thus, the decomposition approach results in simpler reduced-order test systems.

To the authors' best knowledge, the problem of decentralized H_∞ controller design with multiple delay feedback as well as the problem of fault tolerance for this class of symmetric composite systems has not yet been solved.

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E-mail addresses: bakule@utia.cas.cz (L. Bakule), rehakb@utia.cas.cz (B. Rehák), papik@utia.cas.cz (M. Papík).

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1.2. Outline of the paper

Section 2 contains the state space description of the system in an overall form as well as the structure of feedback which is summarized in the problem statement. In Section 3, a single delay controller design as well as its extension to a multiple delay controller design is presented including computation algorithms. This section contains the fault-tolerant analysis resulting in simply verifiable test conditions based on certain systems of reduced dimensions. In Section 4, a numerical example illustrates the potential of the presented methodology.

2. Problem statement

2.1. System description

Structured system consists of N interconnected subsystems, where the ith subsystem has the form

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Gw_i(t) + \sum_{i\neq j,\ j=1}^{N} Hx_j(t), \quad z_i(t) = Cx_i(t), \quad i = 1, \dots, N,\ N > 2, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the subsystem state, $u_i(t) \in \mathbb{R}^m$ is the control input, $w_i(t \in \mathbb{R}^p)$ is the exogenous disturbance signal belonging to $\mathcal{L}_2[0, \infty)$, and $z_i(t) \in \mathbb{R}^q$ is the penalty. A, B, G, H, C are constant matrices of appropriate dimensions.

Overall system description of the system (1) is

$$S^g : \begin{aligned} \dot{x}(t) &= A^g x(t) + B^g u(t) + G^g w(t), \\ z(t) &= C^g x(t), \end{aligned} \tag{2}$$

where $x = (x_1^T, \dots, x_N^T)^T$, $u = (u_1^T, \dots, u_N^T)^T$, $w = (w_1^T, \dots, w_N^T)^T$, and $z = (z_1^T, \dots, z_N^T)^T$ are the states, inputs, disturbances, and penalties, respectively. The matrices are defined as $A^g = (A^g_{ij})$, $A^g_{ii} = A$, $A^g_{ij} = H$ for $i \neq j$, $B^g = \mathrm{diag}(B, \dots, B)$, $G^g = \mathrm{diag}(G, \dots, G)$, and $C^g = \mathrm{diag}(C, \dots, C)$.

2.2. Delayed feedback

Decentralized H_∞ state control supposes that the states in the feedback are delayed. Arbitrary time-varying delays acting within a given bounded interval are considered in local loops. Suppose the controller for the structured system (1) as

$$u_i(t) = Kx_i(t - \tau_i(t)) \tag{3}$$

with the bounds

$$0 \le \tau_i(t) \le \overline{\tau}, \quad i = 1, \dots, N, \tag{4}$$

where $\overline{\tau}$ is a given positive constant. The control (3) can be equivalently written for the overall system (2) as

$$u(t) = \sum_{i=1}^{N} D_i K^g E_i x_i(t - \tau_i(t)), \tag{5}$$

where $K^g = \mathrm{diag}(K, \dots, K)$, $D_i = \mathrm{diag}(0, \dots, 0, I, 0, \dots, 0)$, and $E_i = \mathrm{diag}(0, \dots, 0, I, 0, \dots, 0)$. The matrices D_i and E_i are partitioned into N blocks of identical dimensions, where I denotes the $n \times n$ and $m \times m$ identity matrices located at the ith position of the matrices D_i and E_i, respectively.

Fault tolerance H_∞ analysis means to guarantee the asymptotic stability with disturbance attenuation γ of the overall closed-loop system (2), (5) under l local feedback channels failures. By exploiting the particular structure of symmetric systems, the stability test guaranteeing the disturbance attenuation level γ can be performed easily. It leads to a certain reduced-order control design system. Consider the overall closed-loop system as

$$S^c : \begin{aligned} \dot{x}^c(t) &= A^g x^c(t) + \sum_{i=1}^{N} B^g D_i K^g E_i x^c(t - \tau_i(t)) + G^g w(t), \\ z(t) &= C^g x(t), \qquad x^c(t_o) = \Phi^c(t_o), \quad t_o \in [-\overline{\tau}, 0], \end{aligned} \tag{6}$$

where $\Phi^c(t_o)$ denotes the function of initial conditions. Suppose that l channels of the system (6) totally fail, where $l \in \{1, \dots, N\}$ during a certain time interval. The dynamics of actuator failures can be modeled, without any loss of generality, by a generic model as follows

$$S^f : \begin{aligned} \dot{x}^f(t) &= A^g x^f(t) + \sum_{i=l+1}^{N} B^g D_i K^g E_i x^f(t - \tau_i(t)) + G^g w(t), \\ z^f(t) &= C^g x^f(t), \qquad x^f(t_o) = \Phi^f(t_o), \quad t_o \in [-\overline{\tau}, 0] \end{aligned} \tag{7}$$

with the first l failed channels. To simplify the notation, the indices of x^g for l failed loops are dropped in (7).

2.3. The problem

Given the symmetric complex system (2) and the controller (5). The two goals are formulated as follows:

- Decentralized H_∞ state control design
- Fault tolerance H_∞ analysis

The first goal means to derive the method for the gain matrix K selection so that the H_∞ controller (5) globally asymptotically stabilizes the closed-loop system (6) for a certain valid domain for the delays. The second goal means to derive the conditions for the H_∞ fault tolerance of the closed-loop system (7), when a subset l of local controller fails.

3. Main results

The solution attempts to employ the structure of a class of symmetrical systems for the stabilizing decentralized controller as well as effectively computed fault tolerance bounds of the overall closed-loop system. First, the gain matrix selection for the design model of reduced dimension with a single delay is introduced. Then, it is shown how to use this result for a multiple delay decentralized controller. Finally, a simple test for a fault tolerance of the closed-loop system is derived.

3.1. Decentralized H_∞ state feedback control

The design of controller (5) is decomposed into three subsequent steps. Step 1 presents a reduced-order construction of the design system. Step 2 is a proper selection of the gain matrix K. Step 3 concludes the procedure by the implementation of the gain matrix into the overall system (2).

Step 1. The construction of the reduced-order design system is available in detail in Bakule et al. (2013a,b). This is enabled through appropriate transformation of states. Define the matrices $A_s = A - H$ and $A_o = A - (N - 1)H$. The resulting n-dimensional design system has the form

$$S_m : \begin{aligned} \dot{x}_m(t) &= (A_m + \Delta A_m(t))x_m(t) + Bu_m(t) + Gw_m(t), \\ z_m(t) &= Cx_m(t), \end{aligned} \tag{8}$$

where $A_m = A + (\frac{N}{2} - 1)H = \frac{1}{2}(A_o + A_s)$. The uncertainty has the form $\Delta A_m(t) = e(t)\frac{N}{2}H = e(t)\frac{1}{2}(A_o - A_s) = Ue(t)V$ for all $e(t) \in [-1, 1]$. U and V are arbitrarily factorized matrices.

Step 2. Consider a single delay controller for the system (9)

$$u_m(t) = Kx_m(t - \tau_m(t)), \tag{9}$$

where $0 \le \tau_m(t) \le \overline{\tau}$. The robust version of Theorem 1 by Peng and Tian (2007) follows. It is required for the selection of the gain K with an H_∞ norm bound γ due to the uncertainty term $\Delta A_m(t)$ in the system (8).