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Variational model with kernel metric-based data term for noisy image segmentation

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The segmentation of images with severe noise has always been a very challenging task because noise has great influence on the accuracy of segmentation. This paper proposes a robust variational level set model for image segmentation, involving the kernel metric based on the Gaussian radial basis function (GRBF) kernel as the data fidelity metric. The kernel metric can adaptively emphasize the contribution of pixels close to the mean intensity value inside (or outside) the evolving curve and so reduce the influence of noise. We prove that the proposed energy functional is strictly convex and has a unique global minimizer in $BV(\Omega)$. A three-step time-splitting scheme, in which the evolution equation is decomposed into two linear differential equations and a nonlinear differential equation, is developed to numerically solve the proposed model efficiently. Experimental results show that the proposed method is very robust to some types of noise (namely, salt & pepper noise, Gaussian noise and mixed noise) and has better performance than six state-of-the-art related models.

1. Introduction

Image segmentation is a key initial step which facilitates the subsequent tasks such as image analysis and pattern recognition. For a given image, the goal of segmentation is to partition the image domain into two or more dissimilar regions, each representing an object. However, most of natural images could be degraded by noise during acquisition and transmission. Noise has great influence on the accuracy of segmentation, thus segmentation for noisy images has always been a very challenging task.

To perform the image segmentation task, many successful methods including variational level set models have been presented in the literature (e.g., [1–15]). Variational level set models perform the segmentation task by minimizing an energy functional defined over a space of level set functions. The energy functional typically consists of data term (external energy) that drives the evolving curve toward the desired locations and regularization term (internal energy) that smoothens the level set function.

The data term (in region-based models) is usually defined via certain data fidelity metric (e.g., L^2-norm metric, a common data fidelity metric), depending on the statistical features of images; for example, from the viewpoint of statistical modeling, the L^2-norm metric can be considered to draw from the Bayesian maximum a posteriori (MAP) estimation of the Gaussian distribution that the noise occurring in the image follows [5,6]. For noisy image segmentation, the proper selection of data fidelity metric depends on the type of noise. In most cases, it is (often implicitly) assumed that the image is degraded by additive Gaussian noise; therefore, the L^2-norm metric has been widely used for the data term in variational level set models (e.g., [1,7–11]) since Chan and Vese [1] primarily presented the quite widely used the "active contour without edges" model. Because the L^2-norm metric has difficulty in handling intensity inhomogeneity, the local weighted L^2-norm metric is adopted in some variational level set models (e.g., [13–15]). By utilizing the local weighted L^2-norm metric instead of the L^2-norm metric, the models in [13–15] can efficiently segment images with intensity inhomogeneity. For images with salt & pepper noise, Jung et al. [16] utilized the L^1-norm metric as the metric of data fidelity. For images with Poisson noise, Lee and Le [17] replaced the L^2-norm metric in the Chan–Vese model [1] by $\int_\Omega (u - f\log(u))d\mathbf{x}$, where u is the piecewise constant function related to level set function. The data fidelity metrics mentioned above can handle specific noise, but they may be only appropriate for one type of noise. Recently, the kernel metric has been studied in the literature (e.g., [18–20]). Inspired by the kernel metric, Wu et al. [21] adopted the kernel metric based on the Gaussian radial basis function (GRBF) kernel to define the data term. This kernel metric can adaptively emphasize the contribution of pixels close

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to the mean intensity value inside (or outside) the evolving curve and so reduce the influence of noise. Moreover, this kernel metric is a quite flexible alternative to several metrics (e.g., L^2-norm metric, L^1-norm metric), and can be appropriate for some different types of noise.

The regularization term is usually utilized to restrain the oscillation of level set function and smoothen the evolving curve. In [1], Chan and Vese employed the length of curve as the regularization term. In [7], Chan et al. introduced the TV functional as the regularization term to obtain the global convex model (GCV). Based on the GCV model, Bresson et al. [8] defined the regularization term by integrating an edge indicator function into the TV functional. In [22], Zhang et al. presented an extraordinary reaction–diffusion scheme, in which the diffusion term is originated from the H^1 regularization. Wu and He [9] employed the H^1 regularization to regularize the auxiliary function instead of level set function. Zhang et al. [23,24] directly used the Gaussian filter to act on the level set function at each iteration to play the role of regularization term.

In the paper, we propose a variational level set model with kernel metric for image segmentation, in which the data term is defined by the kernel metric based on the GRBF kernel and the regularization term is defined by the TV functional. We prove that the proposed model is strictly convex and has a unique minimizer in $BV(\Omega)$. Due to the properties that the Ginzburg–Landau (GL) functional Γ-converges to the TV functional [25,26] and can be efficiently solved by the MBO scheme [27], we replace the TV functional with the GL functional in the numerical implementation and design a three-step time-splitting scheme, in which the evolution equation is decomposed into a linear differential equation, a linear diffusion equation and a nonlinear differential equation, respectively. Experimental results show that the proposed method is very robust to some types of noise (i.e. salt & pepper noise, Gaussian noise and mixed noise) and has better performance compared to some related models.

The remainder of this paper is organized as follows. In Section 2, we review some related models. In Section 3, we describe the proposed model and present some theoretic results. In Section 4, we give the numerical implementation. The experimental results are presented in Section 5 and the conclusions are given in Section 6.

2. Related works

2.1. Chan–Vese model

The Mumford–Shah model [28] is a general mathematical model that can achieve image segmentation goal. The basic idea is to seek a pair (u, Γ) for a given image $I : \Omega \subset R^2 \to R$, such that $u : \Omega \to R$ is a piecewise smooth approximation of the image I and the curve Γ is a set of edges between non-overlapping subregions within the image I. In detail, the Mumford–Shah model is the minimization problem of the following energy functional:

$$E(u,\Gamma) = \lambda \int_{\Omega} (I-u)^2 d\mathbf{x} + \int_{\Omega\backslash\Gamma} |\nabla u|^2 d\mathbf{x} + \mu \cdot Length(\Gamma), \tag{1}$$

where $\lambda > 0$ and $\mu > 0$ are fixed scale parameters, ∇ is the gradient operator and $Length(\Gamma)$ is the length of the curve Γ.

Since the minimization problem of the Mumford–Shah functional (1) is formidable, it is a useful simplification of (1) to approximate the intensities inside each of the non-overlapping subregions by a constant. For the two-phase piecewise constant case, where $\nabla u = 0$ in each subregion, the Mumford–Shah functional (1) is simplified to

$$E(c_1,c_2,\Gamma) = \lambda \int_{inside(\Gamma)} (I-c_1)^2 d\mathbf{x} + \lambda \int_{outside(\Gamma)} (I-c_2)^2 d\mathbf{x} + \mu \cdot Length(\Gamma), \tag{2}$$

where c_1 and c_2 are constants depending on Γ.

In the level set method proposed by Osher and Sethian [29], the unknown curve Γ can be represented by the zero level set of a Lipschitz function $\phi(\mathbf{x})$ with the following properties:

$$\begin{cases} \Gamma = \{\mathbf{x} \in \Omega : \phi(\mathbf{x}) = 0\}, \\ inside(\Gamma) = \{\mathbf{x} \in \Omega : \phi(\mathbf{x}) > 0\}, \\ outside(\Gamma) = \{\mathbf{x} \in \Omega : \phi(\mathbf{x}) < 0\}. \end{cases} \tag{3}$$

The function $\phi(\mathbf{x})$ with the above properties is called level set function.

Based on (2) and (3), Chan and Vese [1] proposed the following energy functional in the level set formulation:

$$\begin{aligned} E(c_1,c_2,\phi) = \lambda_1 \int_{\Omega} (I-c_1)^2 H(\phi) d\mathbf{x} \\ + \lambda_2 \int_{\Omega} (I-c_2)^2 (1-H(\phi)) d\mathbf{x} \\ + \mu \int_{\Omega} \delta(\phi)|\nabla\phi| d\mathbf{x}, \end{aligned} \tag{4}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$ and $\mu \geq 0$ are fixed scale parameters, $H(\cdot)$ and $\delta(\cdot)$ denote the one-dimensional Heaviside function and Dirac function, respectively. In practice, they used the slightly regularized versions $H_\varepsilon(\cdot)$ and $\delta_\varepsilon(\cdot)$ instead of $H(\cdot)$ and $\delta(\cdot)$ in the energy functional (4).

Chan and Vese [1] employed the alternating minimization scheme to solve the minimization problem of the energy functional (4) with respect to both (c_1, c_2) and ϕ, in which the steepest descent method is used for the solution of the minimization problem of the energy functional (4) with respect to ϕ and an implicit finite difference scheme is designed to solve numerically the solution of the associated Euler–Lagrange equation. When the solution of the difference equation (denoted by ϕ^n) comes to a steady state, the zero level set of ϕ^n could become the contours that separate the objects from the background.

Due to adopt the region information of image, the Chan–Vese model behaves well in detecting objects with weak or discontinuous edges. However, the non-convexity of the energy functional (4) with respect to ϕ could make it trap in local minima; this could lead to poor segmentation if the initial contours are not chosen properly. In addition, the adoption of L^2-norm for the definition of data term makes the Chan–Vese model is not robust to severe noise.

2.2. Fuzzy active contour model with kernel metric

Krinidis and Chatzis [30] proposed the fuzzy energy-based active contour model, combining fuzzy membership function and the Mumford–Shah functional (2). Different from the Chan–Vese model, this model implicitly represents the evolving curve Γ by the 0.5-level set of a fuzzy membership function u with the following properties:

$$\begin{cases} \Gamma = \{\mathbf{x} \in \Omega : u(\mathbf{x}) = 0.5\}, \\ inside(\Gamma) = \{\mathbf{x} \in \Omega : u(\mathbf{x}) > 0.5\}, \\ outside(\Gamma) = \{\mathbf{x} \in \Omega : u(\mathbf{x}) < 0.5\}. \end{cases} \tag{5}$$

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