Automatica 67 (2016) 185–191

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

State and parameter estimation in 1-D hyperbolic PDEs based on an adjoint method[☆]

Van Tri Nguyen[a], Didier Georges[a], Gildas Besançon[a,b]

[a] Univ. Grenoble Alpes, GIPSA-Lab, F-38000 Grenoble, France

[b] Institut Universitaire de France, France

ARTICLE INFO

Article history:
Received 3 June 2015
Received in revised form
16 November 2015
Accepted 11 December 2015
Available online 5 February 2016

Keywords:
State estimation
Parameter estimation
Hyperbolic system
Adjoint method
Inverse problem
Saint-Venant model
Lighthill–Whitham–Richards model

ABSTRACT

An optimal estimation method for state and distributed parameters in 1-D hyperbolic system based on adjoint method is proposed in this paper. A general form of the partial differential equations governing the dynamics of system is first introduced. In this equation, the initial condition or state variable as well as some empirical parameters are supposed to be unknown and need to be estimated. The Lagrangian multiplier method is used to connect the dynamics of the system and the cost function defined as the least square error between the simulation values and the measurements. The adjoint state method is applied to the objective functional in order to get the adjoint system and the gradients with respect to parameters and initial state. The objective functional is minimized by Broyden–Fletcher–Goldfarb–Shanno (BFGS) method. Due to the non-linearity of both direct and adjoint system, the nonlinear explicit Lax–Wendroff scheme is used to solve them numerically. The presented optimal estimation approach is validated by two illustrative examples, the first one about state and parameter estimation in a traffic flow, and the second one in an overland flow system.

1. Introduction

In nature and ordinary life, one can find a lot of physical laws described by hyperbolic partial differential equations of order one such as water flow, traffic flow, gas dynamics or electrical lines for instance. Researchers in control have investigated a large number of problems concerning this type of systems with different purposes for example to design an infinite-dimensional nonlinear predictive control for open-channel flow (Georges, 2009); to model and control a dam-river system (Litrico & Georges, 1999a,b); to investigate the receding horizon boundary control applied on Lighthill–Whitham–Richards traffic flow model to avoid shock waves (Pham, Georges, & Besançon, 2012); to optimally control traffic in highway network using linear programming (Li, Canepa, & Claudel, 2014); to apply the predictive control method on gas jet flames distribution (Sun & Wang, 2005).

One of the important problems arising in the simulation and control of hyperbolic systems is that uncertainty of the initial condition and empirical parameters can cause large errors and inconsistency between the output of control system and the real one. This motivated few studies on observer design, and even output feedback control, going back to works of Christofides and Daoutidis (1996, 1998) for instance, with pole placement and Kalman designs, up to Hasan (2014) more recently, with backstepping approaches (as also in Vazquez, Krstic, and Coron (2011)). In the present paper, the purpose is to develop a method based on optimal control theory to optimally estimate the initial condition and distributed parameters in such 1-D hyperbolic systems. Some authors studied constant parameter estimation in hyperbolic system such as Becker et al. who used the method of influence coefficient to estimate Manning roughness coefficient in an unsteady open-channel flow (Becker & Yeh, 1972); H. Longxi investigated a complex method to estimate the values of all roughness of a channel network (Longxi, 2008); Y. Ding et al. proposed an adjoint analysis method to find out roughness coefficient in shallow water (Ding, Jia, & Wang,

☆ This work has been partially supported by the LabEx PERSYVAL-Lab (ANR-11-LABX-0025-01) and the MEPIERA project, Grenoble Institute of Technology. The material in this paper was partially presented at the 2014 IEEE Multi-Conference on Systems and Control (MSC 2014), October 8–10, 2014, Antibes, France, at the 2015 American Control Conference (ACC 2015), July 1–3, 2015, Chicago, IL, USA and at the 2015 European Control Conference (ECC 2015), July 15–17, 2015, Linz, Austria. This paper was recommended for publication in revised form by Associate Editor Xiaobo Tan under the direction of Editor Miroslav Krstic.

E-mail addresses: van-tri.nguyen@gipsa-lab.grenoble-inp.fr (V.T. Nguyen), didier.georges@gipsa-lab.grenoble-inp.fr (D. Georges), gildas.besancon@gipsa-lab.grenoble-inp.fr (G. Besançon).

http://dx.doi.org/10.1016/j.automatica.2016.01.031

2004). In the case of distributed parameters, Y. Ding et al. also considered the same approach and applied it on the multi-reaches channel flow network (Ding & Wang, 2005); Richard et al. considered a numerical scheme used to solve parameter identification issue for 1-D hyperbolic system (Ewing & Lin, 1988); whereas Wenhuan investigated a quasi-Newton method to deal with the same problem (Yu, 1999). In a former paper of Bagchi and ten Brummelhuis (1990), the parameter and system state of a discrete-time hyperbolic system with noisy boundary condition are estimated simultaneously based on the maximization of a likelihood function. More recently, we proposed an adjoint method to estimate the initial conditions in an overland flow described by a one dimensional Saint-Venant equation (Nguyen, Georges, & Besancon, 2014). Notice that contemporaneously with it, Hasan et al. investigated a moving horizon technique to estimate state and constant parameter in a 2×2 linear hyperbolic system based on a distributed model for drilling application (Hasan & Imsland, 2014). In that study, the model was discretized first and the adjoint method was applied to the resulting finite dimensional system. In the present paper, which can be considered as an extended version of Nguyen et al. (2014) to general hyperbolic systems, the adjoint method is formulated and solved directly on nonlinear infinite dimensional models. More precisely, we deal with a parameter and state estimation approach in a one-dimensional nonlinear hyperbolic system, with a variable denoted by $u(x,t)$, and a flow denoted by $f(u(x,t),x)$, respectively depending on some initial condition denoted by $u_0^i(x)$, and some distributed parameters denoted by $\alpha_i(x)$, both supposed to be unknown, and thus needing to be estimated. The adjoint analysis is formulated with the original infinite dimensional system, to connect the sensitivity of variables needing to be estimated with the system model and the measurements. These points are also the main contributions of the present paper, and to the best of our knowledge, there are very few researches realized with such a spirit.

The rest of this paper is organized as follows: Section 2 describes the dynamics of system and the formulation of optimal estimation problem. The adjoint method is applied to the optimization problem to get the adjoint system and gradient of estimated variables is presented in Section 3. In Section 4, two illustrative examples dealing with parameter and state estimation in traffic flow and overland flow system are presented. Some conclusions and perspectives are given at the end of the paper.

2. Estimation problem statement

2.1. System dynamics

Let us consider a general form of 1-D hyperbolic system of variable $u(x,t)$ and flow $f(u(x,t),x)$ a function of $u(x,t)$ and x. The spatial variable x and time variable t belong to the set $(x,t) \in [0,L] \times \mathbb{R}^+$, and the system reads:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} + \dfrac{\partial f(u(x,t),x)}{\partial x} = g(x,t) \\ u(x,0) = u_0^i(x) \\ u(0,t) = u_0^b(t) \end{cases} \tag{1}$$

where the function $u_0^b(t)$ is a predefined boundary condition, the function $u_0^i(x)$ denotes the initial condition, and $g(x,t)$ is a known function. Notice that if it is clear enough in the sequel, notation f and u will be used instead of $f(u(x,t),x)$ and $u(x,t)$ in order to shorten the equation length. Let us assume, without any loss of generality, that the function $f(u,x)$ can be written in the following form for some vectors $\alpha = [\alpha_1(x) \dots \alpha_i(x) \dots \alpha_K(x)]^T$ and $\varphi = [\varphi_i(u) \dots \varphi_i(u) \dots \varphi_K(u)]^T$.

$$f = \sum_{i=1}^{K} \alpha_i(x)\varphi_i(u). \tag{2}$$

On this basis, let us consider the problem of estimating time and space evolution of u when initial condition $u_0^i(x)$ is unknown, together with function parameter $\alpha(x)$. Once the initial state and parameter of system are successfully recovered, all transient state profiles of the system can be fully rebuilt by simulation.

2.2. Optimal estimation problem

For the estimation problem of parameter and state in the considered hyperbolic system, one can use two main approaches: empirical procedures or minimization approach based on optimal control theory. The first approach uses direct empirical formula with observation data to get the parameter value and is suitable only for simple parameter estimation. In the present work, due to the complexity and non linearity of estimation problem, we use the second approach to minimize the errors between simulations and some lumped observation values of variable $u(x,t)$. In other words, we minimize a cost function J defined as follows:

$$\begin{aligned} J = {} & \frac{1}{2}\sum_{j=1}^{N}\int_0^T \left\{ \int_0^L \delta_A(x-x_j)u\,dx - u_j^{meas}(x_j,t) \right\}^2 dt \\ & + \frac{1}{2}\varepsilon_1 \int_0^L \|u_0^i(x) - u_{0F}^i(x)\|^2 dx \\ & + \frac{1}{2}\varepsilon_2 \sum_{i=1}^{K}\int_0^L \|\alpha_i(x) - \alpha_{iF}(x)\|^2 dx \end{aligned} \tag{3}$$

where T = optimization horizon (*hours*); L = considered spacial length where the system takes place (m); N = number of observation values of $u(x,t)$; $u_j^{meas}(x_j,t)$ = measured value of $u(x,t)$ at observation position x_j with $x_j \in [0,L]$; $\alpha_{iF}(x)$ = first guessed values of parameters; u_{oF}^i = first guessed values of initial condition; ε_1 and ε_2 = weighting factor applied to first guessing term to calibrate the estimated value and the guessed one and adjust the scale of objective function. The term $\delta_A(x-x_j)$ is an approximation of Delta-Dirac function described by a Gaussian function with a very small variance σ^2 as $\delta_A(x-x_j) = e^{-(x-x_j)^2/\sigma^2}$.

3. Adjoint-based approach

3.1. Variational analysis

From the previous analysis, the optimal values of state and parameter must minimize the cost function in Eq. (3) and satisfy also the system Eq. (1). This constraint and the continuity of the first partial derivative of both system dynamics and cost function lead to use the Lagrange multiplier with Lagrangian variable $\lambda(x,t)$, which allows to combine the system equation and cost function into only one new cost functional $\mathcal{L}(\lambda,\rho,\alpha)$, shortened to $\mathcal{L}$, as follows.

$$\mathcal{L} = J + \underbrace{\int_0^T \int_0^L \lambda \left[\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} - g(x,t) \right] dxdt}_{\mathcal{A}}. \tag{4}$$

To deal with this optimization problem the common adjoint method is used in order to obtain the adjoint system and establish the gradient of cost functional with respect to the parameters and state needing to be estimated. These gradients describe the