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Brief paper

Striped Parameterized Tube Model Predictive Control[☆]

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ABSTRACT

A modification of the Parameterized Tube Model Predictive Control (PTMPC) strategy for linear systems with additive disturbances is proposed, which reduces the dependence of the number of optimization variables on horizon length from quadratic to linear by using a triangular striped prediction structure. Unlike PTMPC, which assumes a fixed linear terminal feedback law for predictions, the proposed prediction scheme allows disturbance compensation to extend beyond the initial N-step prediction horizon. The resulting scheme can potentially outperform PTMPC in terms of the size of the domain of attraction and allows for a longer horizon N for the same computational demand.

1. Introduction

Robust model predictive control (RMPC) of linear systems subject to additive uncertainty is an important area of research. Optimal RMPC requires the solution of dynamic programming (Bertsekas, 1995) or minimax problems (Scokaert & Mayne, 1998) and is computationally intractable, thus a compromise between sub-optimality and complexity is necessary. Early RMPC considered open-loop solutions, where a single input sequence is determined for all disturbance realizations. This is conservative because it ignores information on future states and uncertainties that will be available for use by the controller and can thus lead to poor performance and infeasibility. Quasi-closed loop formulations on the other hand optimize perturbations on a pre-stabilizing law (Langson, Chryssochoos, Raković, & Mayne, 2004; Lee & Kouvaritakis, 1999). They provide an improvement, but are still conservative because pre-stabilization is designed offline.

Optimality is improved upon by affine-in-the-disturbance MPC (ADMPC) (Goulart, Kerrigan, & Maciejowski, 2006; Lofberg, 2003). This uses feedforward plus linear disturbance compensation with a triangular structure in the near horizon, while in the far horizon it deploys fixed state feedback. ADMPC has been superseded by parameterized tube MPC (PTMPC) (Raković, Kouvaritakis, Cannon, Panos, & Findeisen, 2012) which exploits a separable triangular prediction structure of partial tubes, the first of which describes the nominal dynamics and the rest are associated with future disturbances. These are unknown and are defined in terms of the vertices of the allowable set of disturbances. The PTMPC policy is piecewise-affine-in-the-disturbance and hence leads to larger domains of attraction than ADMPC. The number of variables and constraints grows quadratically with the prediction horizon N in both ADMPC and PTMPC which therefore are limited to low-dimensional systems or small N.

Here we present a RMPC formulation that is a modification of PTMPC in which the degrees of freedom affect (directly) the inputs over the entire prediction horizon with a striped structure. This idea has been explored before but in the context of constructing parameterized robust control invariant sets through the use of a sequence contracting technique (Raković & Baric, 2010). It has also been explored in Stochastic MPC (Kouvaritakis, Cannon, & Muñoz-Carpintero, 2013), where affine-in-the-disturbance compensation extends over the infinite far prediction horizon according to a fixed control law computed offline. In our approach the disturbance compensation in the far horizon is computed online, and leads to a terminal control law more general than linear feedback. This is achieved by expressing predictions as the sum of a nominal sequence and a single sequence associated with all particular future disturbances. This gives rise to a separable striped disturbance compensation scheme which is allowed to extend

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over an infinite prediction horizon and leads to a number of variables and constraints which grows linearly with N (rather than quadratically as for PTMPC). Additionally, allowing for disturbance compensation into the far horizon implies a constraint relaxation. Simulations show that this strategy can, for a comparable number of degrees of freedom and constraints, lead to larger domains of attraction.

Section 2 gives the system description and a brief review of the separable scheme of Raković et al. (2012). Our strategy is introduced in Section 3, and Section 4 analyzes the control theoretic properties of two variants of the strategy: one with exponential convergence to a known minimal robust invariant set; and another based on input-to-state stability, as presented in Muñoz-Carpintero, Kouvaritakis, and Cannon (2014), which enjoys increased control authority but does not have the guarantee of convergence to the minimal robust invariant set. Section 5 presents an illustration by simulation of the benefits of the proposed strategy and conclusions are drawn in Section 6.

Notation: $\mathbb{N}_+$ and $\mathbb{R}_+$ denote the sets of positive integers and positive reals. $\mathbb{N} = \mathbb{N}_+ \cup \{0\}$ and $\mathbb{N}_{[a,b]} = \{a, a+1, \ldots, b\}$. For $X, Y \subset \mathbb{R}^n$, $X \oplus Y = \{x + y : x \in X, y \in Y\}$ denotes the Minkowski sum, and the image of X under $M \in \mathbb{R}^{m\times n}$ is $MX = \{Mx : x \in X\}$. For $X = \mathrm{conv}(\{x_1, \ldots, x_n\})$ (where $\mathrm{conv}(\cdot)$ denotes the convex hull) and $A, B \in \mathbb{R}^{m\times n}$, $(A, B)X = \mathrm{conv}(\{(Ax, Bx) : x \in X\})$. For $Y = \{y : E_l y \le 1,\ l \in \mathbb{N}_{[1,n_y]}\}$, the Y-distance function of x is $\mathrm{dist}_Y(x) = \max(\{E_l x - 1 : l \in \mathbb{N}_{[1,n_y]}\} \cup \{0\})$, and the maximum Y-distance function of X is $\mathrm{maxdist}_Y(X) = \max(\{E_l x - 1 : x \in X,\ l \in \mathbb{N}_{[1,n_y]}\} \cup \{0\})$.

2. System description and separable prediction scheme

Consider the linear, discrete-time system and constraints

$$x^+ = Ax + Bu + w, \tag{1}$$

$$Fx + Gu \le 1, \tag{2}$$

with $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $w \in \mathbb{W} \subset \mathbb{R}^{n_x}$. $F \in \mathbb{R}^{n_c\times n_x}$ and $G \in \mathbb{R}^{n_c\times n_u}$. Here $\mathbb{W} = \mathrm{conv}\left(\{\tilde{w}_i : i \in \mathbb{N}_{[1,q]}\}\right)$ is a polytope that contains the origin and $\mathbb{Y} = \{y = (x, u) :\ F_l x + G_l u \le 1, l \in \mathbb{N}_{[1,n_c]}\}$ is a polytope that contains the origin in its interior; the subscript l denotes the lth row.

A robust MPC strategy with a separable prediction scheme is presented in Raković et al. (2012) in which, since the system is linear, the predictions can be split in sequences, one associated with the nominal dynamics ($w \equiv 0$) and the rest associated with each future disturbance. The 0th partial state and control sequences $\mathbf{x}_{(0,:)} = \{x_{(0,k)}\}_{k\in\mathbb{N}}$ and $\mathbf{u}_{(0,:)} = \{u_{(0,k)}\}_{k\in\mathbb{N}}$ account for the nominal dynamics satisfying at each prediction time $k \in \mathbb{N}$

$$x_{(0,k+1)} = Ax_{(0,k)} + Bu_{(0,k)}, \quad \text{with } x_{(0,0)} = x, \tag{3}$$

where x is the current state. Similarly, the dynamical contribution of the disturbance acting at the $(j-1)$th prediction time, w_{j-1}, $j \in \mathbb{N}_+$ is given by the jth partial state and control sequences, $\mathbf{x}_{(j,:)} = \left\{x_{(j,k)}\right\}_{k\in\mathbb{N}_{[j,\infty)}}$ and $\mathbf{u}_{(j,:)} = \left\{u_{(j,k)}\right\}_{k\in\mathbb{N}_{[j,\infty)}}$, which for $k \in \mathbb{N}_{[j,\infty)}$ satisfy

$$x_{(j,k+1)} = Ax_{(j,k)} + Bu_{(j,k)}, \quad \text{with } x_{(j,j)} = w_{j-1}. \tag{4}$$

Since w_{j-1} is unknown, the predictions of $x_{(j,k)}$ and $u_{(j,k)}$ are also unknown, however it is possible to find exactly the sets that contain them. Considering the extreme realizations, $\tilde{w}_i$, $i \in \mathbb{N}_{[1,q]}$, of w_{j-1}, and denoting corresponding jth partial extreme state and control sequences as $\mathbf{x}_{(i,j,:)} = \{x_{(i,j,k)}\}_{k\in\mathbb{N}_{[j,\infty)}}$ and $\mathbf{u}_{(i,j,:)} = \{u_{(i,j,k)}\}_{k\in\mathbb{N}_{[j,\infty)}}$, we define $u_{(j,k)} = \sum_{i=1}^{q} \lambda_i(w_{j-1}) u_{(i,j,k)}$ in terms

Control tubes				Mode 1	Mode 2	
$u_{(0,0)}$	$u_{(0,1)}$	$u_{(0,2)}$	$\cdots$	$u_{(0,N-1)}$	$Kx_{(0,N)}$	$\cdots$
	$U_{(1,1)}$	$U_{(1,2)}$	$\cdots$	$U_{(1,N-1)}$	$KX_{(1,N)}$	$\cdots$
		$U_{(2,2)}$	$\cdots$	$U_{(2,N-1)}$	$KX_{(2,N)}$	$\cdots$
			$\ddots$	$\vdots$	$\vdots$	
				$U_{(N-1,N-1)}$	$KX_{(N-1,N)}$	$\cdots$
					$KX_{(N,N)}$	$\ddots$
u_0	u_1	u_2	$\cdots$	u_{N-1}	u_N	

Fig. 1. Triangular prediction scheme of PTMPC – Inputs.

of convex interpolation parameters $\lambda_i(w_{j-1})$, $i \in \mathbb{N}_{[1,q]}$ such that $w_{j-1} = \sum_{i=1}^{q} \lambda_i(w_{j-1})\tilde{w}_i$, and it follows that

$$\begin{aligned} x_{(j,k)} &\in X_{(j,k)} = \mathrm{conv}\left(\left\{x_{(i,j,k)} : i \in \mathbb{N}_{[1,q]}\right\}\right), \\ u_{(j,k)} &\in U_{(j,k)} = \mathrm{conv}\left(\left\{u_{(i,j,k)} : i \in \mathbb{N}_{[1,q]}\right\}\right), \end{aligned} \tag{5}$$

where the dynamics of the jth partial extreme state and control sequences for $i \in \mathbb{N}_{[1,q]}, j \in \mathbb{N}_+$, are given by

$$x_{(i,j,j)} = \tilde{w}_i, \tag{6a}$$

$$x_{(i,j,k+1)} = Ax_{(i,j,k)} + Bu_{(i,j,k)}, \quad k \in \mathbb{N}_{[j,\infty)}. \tag{6b}$$

Then the full predictions are given by

$$x_k = \sum_{j=0}^{k} x_{(j,k)}, \qquad u_k = \sum_{j=0}^{k} u_{(j,k)}, \quad \forall k \in \mathbb{N} \tag{7}$$

thus defining a triangular prediction structure as shown for the inputs in Fig. 1 (the case for the states is analog), where from (5) and (7), the full predicted inputs are contained in sets given by the Minkowski sum of the elements in each column.

Assumption 1. (i) The pair (A, B) is stabilizable; (ii) The matrix gain K is such that $\Phi = A + BK$ is strictly stable and the minimal robust invariant set of (1) under $u = Kx$, Ω_K^∞, satisfies $(I, K)\Omega_K^\infty \in \mathrm{interior}(\mathbb{Y})$.

A control policy is defined implicitly in this separable prediction scheme. From (7) the predicted inputs can be re-written as $u_k = u_{(0,k)} + \sum_{j=1}^{k} u_{(j,k)}$, where $u_{(j,k)}$ is a convex interpolation of $u_{(i,j,k)}$, $i \in \mathbb{N}_{[1,q]}$ depending on the value of $w_{(j-1)}$. Thus, the associated control policy is piecewise-affine-in-the-disturbance (Raković et al., 2012). Note that there is no need to know the interpolation parameters, as we only need to know the extreme trajectories to guarantee feasibility.

The first N prediction steps are referred to as Mode 1 and the remainder as Mode 2, where it is usual to deploy a stabilizing terminal control law. Then, in Raković et al. (2012), $x_{(0,k)}$, $u_{(0,k)}$, $x_{(i,j,k)}$ and $u_{(i,j,k)}$ are free optimization variables in Mode 1, while in Mode 2 $u_{(0,k)}$ and $u_{(i,j,k)}$ are associated to the terminal control law $u = Kx$ and are given by $u_{(0,k)} = Kx_{(0,k)}$ and $u_{(i,j,k)} = Kx_{(i,j,k)}$ (see Fig. 1). On the other hand, a striped structure to reduce the number of variables and constraints is invoked in this paper (see Fig. 2). The details are presented next.

3. The RMPC Strategy: Striped prediction scheme and constraints

Consider the following structural constraints in addition to (3)–(7): for $j \in \mathbb{N}_+$, $k \in \mathbb{N}_{[j,\infty)}$, $i \in \mathbb{N}_{[1,q]}$

$$u_{(i,j,k)} := u_{(i,1,k-j+1)}, \qquad x_{(i,j,k)} = x_{(i,1,k-j+1)}, \tag{8}$$

which imply that $X_{(j,k)} = X_{(1,k-j+1)}$ and $U_{(j,k)} = U_{(1,k-j+1)}$, so that the elements on each diagonal below the first row (see Fig. 2) are the same, yielding a striped structure. The 1st partial extreme state and controls $x_{(i,1,k)}$, $u_{(i,1,k)}$ are free optimization variables for $k \in \mathbb{N}_{[1,N]}$ and $k \in \mathbb{N}_{[1,N-1]}$, respectively, and we fix $u_{(i,1,k)} = Kx_{(i,1,k)}$ for $k \in \mathbb{N}_{[N,\infty)}$. Thus all the jth partial sequences are fully defined