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Cost function to analyze the passband and the stopband of Digital Biquadratic Filters

Sergio Ledesma *, Mario-Alberto Ibarra-Manzano, Dora-Luz Almanza-Ojeda, Eduardo Cabal-Yepez

School of Engineering, University of Guanajuato, Salamanca, GTO 36885, Mexico

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ABSTRACT

This paper focuses on filter design in the frequency domain; specifically, the frequency behavior of one type of IIR filter called digital biquadratic filter is analyzed. First, similar mathematical approaches were used to define two normalized functions: one for the stopband and another for the passband. These two functions can be combined to create a cost function, and therefore, used by optimization algorithms to design digital filters. Computer simulations allowed building color maps to analyze this cost function. These maps describe areas that allow understanding visually the effect of the poles and zeros of the filter over its frequency behavior. The areas described by these maps introduce background information that can be used to design or optimize digital filters with custom frequency responses. Several tests were performed to establish the validity of the proposed cost function.
Additionally, it was shown that the cost function can be easily computed using a polynomial approximation. Therefore, instead of using uniformly distributed frequency points, the frequency analysis is performed at the Chebyshev nodes improving accuracy while using very few terms in the approximation. This approximation can be used to speed up the computation of the cost function, and consequently reduce optimization time for complex filters composed of several biquadratic sections.

1. Introduction

In the field of Digital Signal Processing (DSP), a filter is a building block that removes undesired signal components. Digital filters are used for high-speed digital communications, and in systems to process audio and video, see [24] and [30]. There are two main types of digital filters: Infinite Impulse Response (IIR) filters and Finite Impulse Response (FIR) filters. In practice, IIR filters often provide a better performance with less computations than FIR filters, see [28] for more information about the implementation of DSP algorithms and analytical modern procedures. The typical unimodal error surface used to design FIR filters make gradient based algorithms very effective in the design of these filters, [11]. On the other hand, gradient based algorithms may lead to a local minimum when designing IIR filters because the error surface is nonlinear and, in most cases, it is a multi-modal function, see [7]. Additionally, some recursive filters may become unstable and cannot easily provide a linear-phase response as FIR filters can, see [13], [18] and [27].

In recent years, many heuristic optimization design methods have been developed, such as: Simulated Annealing, Genetic Algorithms, Particle Swarm Optimization, etc., see [1], [2], [14] and [25]. One of these heuristic methods is based on Artificial Bee Colony (ABC) algorithms to design adaptive IIR filters, see [11]. ABC algorithms are inspired by the collective behavior of insects and can be used in optimization problems. Karaboga shows that it is possible to used a modified ABC algorithm to design IIR filters, see [10] and [11]. As the error surface for designing an IIR filter is usually nonlinear and may have several local minima, the authors of [3] proposed a seeker-optimization algorithm (SOA) for filter design. They compared the performance of their algorithm with the performance of particle swarm optimization algorithms and genetic algorithms; they conclude that SOA is capable of estimating the filter coefficients for a wide variety of IIR structures. Thus, digital filter design has been the motivation behind the development of several algorithms, see [17] and [21]. For instance, the authors of [16] proposed the use of Differential Evolution to identify the most efficient implementation of a IIR receiver filter for a communication system, see [29]. In the same context, the authors of [12] proposed a method to use particle swarm optimization to design band pass and band stop infinite impulse response filters.

* Corresponding author.
E-mail addresses: selo@ugto.mx (S. Ledesma), ibarram@ugto.mx (M.-A. Ibarra-Manzano), dora.almanza@ugto.mx (D.-L. Almanza-Ojeda), educabal@ugto.mx (E. Cabal-Yepez).

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In medical research, filtering is very important because it is used to analyze signals produced by the body. For instance, the authors of [26] considered different types of digital filters to process electroencephalographic data, in order to reduce distortion and bias in the signals under study. The authors of [23] analyze the application of a Chebyshev approximation to IIR digital filters. In this context, it is clear that the quickly computation of the cost function is very important in the field of digital filter design.

This paper proposes a cost function that is based on polynomial approximation to create color maps to analyze the behavior of IIR filters that are implemented using biquadratic structures. These maps display color areas that can be used to inspect the behavior of the cost function. The main application of this cost function is the design of IIR filters using optimization algorithms. Additionally, we suggest a Chebyshev approximation to speed up the computation of the proposed cost function.

1.1. Biquad filter

In the field of digital signal processing, a digital biquadratic filter is a type of linear filter that has a system function which is the ratio of two quadratic functions. These filters are commonly known as biquad filters, and they contain two poles and two zeros. The system function of a biquad filter is given by

$$H(z) = G\frac{1+\mu_1 z^{-1}+\mu_2 z^{-2}}{1+\lambda_1 z^{-1}+\lambda_2 z^{-2}}, \tag{1}$$

where G is the filter gain, and the constants μ_1, μ_2, λ_1 and λ_2 determine the behavior of the filter. Specifically, its two zeros are defined by the values of μ_1 and μ_2; while its poles are obtained by the values of λ_1 and λ_2. For a biquad filter to be stable, the values of λ_1 and λ_2 are computed so that both poles are inside the unit circle.

Biquad filters are very important because they can be used to implement high-order Infinite Impulse Response (IIR) filters. The main advantage of using a cascade of biquad filters (to implement an IIR filter of high order) is to improve the stability due to the quantization of the coefficients. In fact, previous research indicates that the physical structure of the filter has a big influence on the error introduced by quantization effects and finite coefficient size, see the references in [4] and [5]. Deczky concluded that the cascade and parallel forms have some advantages when compared with the direct form. Essentially, the stability of the cascade form has been thoroughly studied; not to mention that the cascade form is not affected by quantization as much as the direct form is.

2. Proposed method

Computer-aided design of IIR digital filters involves the use of an optimization method to minimize the error between the magnitude of the desired response $|H_d(\omega)|$ and the magnitude of the frequency response $|H(\omega)|$ of the filter. One of the most common error function used in computer-aided design of IIR digital filters is

$$Error = \frac{1}{N}\sum_{k=1}^{N} w(\omega_k)\left[|H(\omega_k)| - |H_d(\omega_k)|\right]^{2p}, \tag{2}$$

where p is a positive integer value and $w(\omega_k)$ is a weighting function. Equation (2) is a convenient and simple cost function to design IIR digital filters. However, it requires the use of a weighting function $w(\omega_k)$. For simplicity, some researches assume that $w(\omega_k) = 1$ for $k = 1, 2, ...N$ and $p = 2$, see [8]. For these values of $w(\omega_k)$ and p, Equation (2) is the Mean Square Error (*MSE*), which is one of the most common error functions used in the field of optimization.

$$MSE = \frac{1}{N}\sum_{k=1}^{N}\left[|H(\omega_k)| - |H_d(\omega_k)|\right]^2 \tag{3}$$

Instead of using the *MSE* of Equation (3), some authors prefer the Root Mean Square Error (*RMSE*) defined as

$$RMSE = \sqrt{MSE}, \tag{4}$$

or the Mean Absolute Error (*MAE*), see [9], defined as

$$MAE = \frac{1}{N}\sum_{k=1}^{N} abs\left(|H(\omega_k)| - |H_d(\omega_k)|\right), \tag{5}$$

where the *abs* function computes the absolute value. These error functions are the most commonly used in IIR filter design, each of them has advantages and disadvantages. Note that these error functions, in general, are special cases of Equation (2) which requires running the optimization algorithm several times. That is, for each weighting function the optimization algorithm is executed, then the actual frequency response of the filter is evaluated. If the response of the filter is acceptable, the design process is completed. However, if the response of the filter is not acceptable, a new weighting function must be proposed and the optimization algorithm must again be executed. Another common problem in Equation (2) is the value of p. For simplicity, some authors use $p = 1$, see [12], but to meet the minimax approximation it is recommended to use at least the sequence: $2p = 2, 4, 10, 40$. The authors of [4] indicated that the largest index they tried was 40, which gave satisfactory results for most of the problems they attempted. This paper proposes a cost function than can be used to design IIR digital filters executing the optimization algorithm only once without the need of adjusting parameters.

The system function of Equation (1) can be used to represent the system function of a cascade of M biquad sections in terms of its zeros and poles, see [19]. Thus, for simplicity of notation, we assume that all poles and zeros occur in complex conjugate pairs so that

$$H(z) = G\prod_{m=1}^{M}\frac{(1-a_m e^{j\alpha_m}z^{-1})(1-a_m e^{-j\alpha_m}z^{-1})}{(1-b_m e^{j\beta_m}z^{-1})(1-b_m e^{-j\beta_m}z^{-1})}, \tag{6}$$

where $a_m e^{j\alpha_m}$ and $a_m e^{-j\alpha_m}$ are the zeros of the system function, while $b_m e^{j\beta_m}$ and $b_m e^{-j\beta_m}$ are its poles. Thus, a_m and b_m are the magnitude of its zeros and poles respectively. In this case, the corresponding phases are denoted by α_m and β_m. If $z = e^{j\omega}$ (see [20]), then,

$$H(\omega) = G\prod_{m=1}^{M}\frac{(1-a_m e^{j\alpha_m}e^{-j\omega})(1-a_m^{-j\alpha_m}e^{-j\omega})}{(1-b_m e^{j\beta_m}e^{-j\omega})(1-b_m^{-j\beta_m}e^{-j\omega})}. \tag{7}$$

The magnitude of the frequency response of this filter can be expressed as

$$|H(\omega)| = |G|\prod_{m=1}^{M}\frac{|1-a_m e^{-j(\omega-\alpha_m)}||1-a_m^{-j(\omega+\alpha_m)}|}{|1-b_m e^{-j(\omega-\beta_m)}||1-b_m^{-j(\omega+\beta_m)}|}, \tag{8}$$

which can be written as

$$|H(\omega)| = |G|\prod_{m=1}^{M}\frac{U(\omega; a_m e^{j\alpha_m})}{U(\omega; b_m e^{j\beta_m})}, \tag{9}$$

where

$$U(\omega; a_m e^{j\alpha_m}) = \sqrt{(1-2a_m\cos(\omega-\alpha_m)+a_m^2)(1-2a_m\cos(\omega+\alpha_m)+a_m^2)}, \tag{10}$$