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Brief paper

Cooperative control of linear multi-agent systems via distributed output regulation and transient synchronization ☆

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ABSTRACT

A wide range of multi-agent coordination problems including reference tracking and disturbance rejection requirements can be formulated as distributed output regulation problem. The general framework captures typical tasks such as output synchronization, leader-following, formation keeping, and many more. We present a distributed regulator for groups of identical and non-identical linear agents subject to global external signals affecting all agents as well as local external signals affecting individual agents. Both signal types comprise references and disturbances. The main contribution is a novel coupling among the agents based on their transient state components, or estimates thereof in the output feedback case. The transient synchronization improves the cooperative behavior in transient phases and guarantees a desired decay rate of the synchronization error, which leads to a cooperative reaction on local disturbances acting on individual agents.

1. Introduction

In a variety of modern man-made systems, it is desirable to synthesize a cooperative behavior among individual dynamical agents, similar to bird flocks and fish schools observed in nature. Examples include multi-vehicle coordination and formation flight problems, robot cooperation in production lines, power balancing in micro-grids, and many more. Of particular interest are distributed control laws which require only local information exchange between neighboring subsystems and no centralized data collection or processing entity, which provides scalable, flexible, and robust algorithms. A fundamental cooperative control problem is the consensus or synchronization problem for groups of linear dynamical agents. It has been studied extensively over the past decade, starting from single-integrator agents (Olfati-Saber & Murray, 2004; Ren & Beard, 2005), to double-integrator agents (Ren & Atkins, 2007), identical linear agents (Fax & Murray, 2004; Li, Duan, Chen, & Huang, 2010; Scardovi & Sepulchre, 2009; Tuna, 2008; Wieland, Kim, & Allgöwer, 2011), and non-identical linear agents (Grip, Yang, Saberi, & Stoorvogel, 2012; Kim, Shim, & Seo, 2011; Lunze, 2012; Wieland, Sepulchre, & Allgöwer, 2011).

From a practical point of view, it is desirable to influence the behavior of the group via external reference signals. A solution to this problem is the leader–follower setup (Hong, Hu, & Gao, 2006; Li et al., 2010; Ni & Cheng, 2010; Zhang, Lewis, & Das, 2011). The idea is to select a particular agent as leader for the group or introduce a virtual leader and design the distributed control law such that all agents synchronize to this leader. Moreover, it is important to consider external disturbances acting on the multi-agent system, to analyze the performance of the closed-loop system, and to incorporate disturbance rejection or attenuation requirements in the design procedure. Rejection of constant disturbances is addressed in Andreasson, Dimarogonas, Sandberg, and Johansson (2014), Seyboth and Allgöwer (2015) and Yucelen and Egerstedt (2012). In order to tackle both reference and disturbance signals simultaneously, it was proposed in Huang (2011) and Xiang, Wei, and Li (2009) to formulate multi-agent coordination problems with external reference and disturbance signals as a *synchronized output regulation* problem and utilize the classical output regulation theory, cf., Huang (2004), Knobloch, Isidori, and Flockerzi (1993) and Trentelman, Stoorvogel, and Hautus (2001). The problem setup in Huang (2011) and

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Xiang et al. (2009) consists of an autonomous exosystem and a group of identical linear agents, which are affected by the generalized disturbance signal generated by the exosystem, and a regulation error for each agent which shall converge to zero. The group objective for the multi-agent system is formulated in terms of a common reference signal for all agents and local regulation errors for each agent with respect to the common reference. Cooperation is necessary since not all agents have access to the external signal. By assumption, the group is divided into a group of informed agents which are able to reconstruct the external signal, and uninformed agents which are dependent on information exchange with informed agents in order to solve their regulation task. The cooperative output regulation problem generalizes existing solutions of the leader–follower problem (Hong, Wang, & Jiang, 2013). Su and Huang (2012a,b) extend the results of Huang (2011), Xiang et al. (2009) and present a solution to the cooperative output regulation problem with non-identical agents based on state feedback and output feedback, respectively. Each agent is described by a generalized plant in which all matrices may be different for different agents. The solution proposed in Su and Huang (2012b) consists of three components: local feedback laws which are constructed based on the classical output regulation theory for each agent; local observers for the state of each agent; and a distributed observer for the generalized disturbance. Besides the distributed observer, this control strategy is decentralized. There are no couplings based on the agent states or outputs and it is inherent in the controller structure that local disturbances are rejected by the affected agent only and no other agent recognizes such a disturbance. This causes the limitation that the group is not able to react in a cooperative manner on local disturbances. In applications such as formation flight and vehicle platooning, keeping a desired formation typically has a higher priority than precisely following a given path with the formation center. A vehicle in a platoon is expected to slow down or accelerate in order to avoid collisions with its follower or predecessor when any of those experiences a disturbance. This fact is also highlighted in Bartels and Werner (2014) and motivates the work in the present paper.

In the present paper, we distinguish between global and local generalized disturbances affecting all or only individual agents in the group, respectively, and present a distributed regulator taking this structure into account. The main contribution is an extension of the distributed regulator with suitable couplings among the agents in order to enable a cooperative reaction of the group on local external disturbances. These couplings allow to impose a desired decay rate on the synchronization error. It is shown that the transient state components of the agents are well suited for this purpose. A recent design method for the coupling gain allows to impose performance specifications such as a minimum decay rate of the synchronization error of the group. Moreover, in a preliminary step, we formulate the cooperative control problem as a single centralized output regulation problem. The solvability conditions for the centralized problem and its particular structure yield necessary and sufficient solvability conditions for the distributed regulation problem.

Outline: Section 2 presents some background and the distributed and centralized output regulation problems. The distributed regulator for general non-identical linear agents is presented in Section 3. In Section 4, the extension of the distributed regulator is derived which guarantees exponential stability of the synchronization error with a desired decay rate. The derivation is based on the assumption that the agents have identical dynamics. A vehicle platooning example illustrates the results. In Section 5, the assumption of identical agent dynamics is relaxed and it is shown how the coupling can be designed for non-identical agents with similar dynamics. Section 6 concludes the paper.

2. Problem setup

Notation: The open left half plane, imaginary axis, and open right half plane of $\mathbb{C}$ are denoted by $\mathbb{C}^-$, $\mathbb{C}^0$, and $\mathbb{C}^+$, respectively. For $z \in \mathbb{C}$, $\bar{z}$ is the complex conjugate, $\mathbf{Re}(z)$ is the real part and $\mathbf{Im}(z)$ is the imaginary part. The spectrum of $A \in \mathbb{C}^{n\times n}$ is denoted by $\sigma(A) \subset \mathbb{C}$. $\mathrm{diag}(M_k) = \mathrm{diag}(M_1, \dots, M_N)$ and $\mathrm{stack}(M_k)$ denote a block diagonal matrix and a vertical stack of matrices with blocks M_k, $k = 1, \dots, N$, respectively. For a set of vectors $v_k \in \mathbb{R}^n$, $k = 1, \dots, N$, $v \in \mathbb{R}^{Nn}$ denotes the stack vector $v = [v_1^\mathsf{T} \ \cdots \ v_N^\mathsf{T}]^\mathsf{T}$. For $v \in \mathbb{R}^n$, $\mathrm{diag}(v)$ is a diagonal matrix with the entries of v on the diagonal. The identity matrix of dimension N is I_N and the vector of ones is $\mathbf{1}$. For a transfer function matrix G, $\|G\|_\infty$ denotes its $\mathcal{H}_\infty$ norm. The symbol $\otimes$ denotes the Kronecker product. The pairs (A, B), (A, C) are called stabilizable, detectable with decay rate $\gamma > 0$, if the decay rate of the uncontrollable, unobservable modes is at least γ, i.e., $(A + \gamma I_n, B)$, $(A + \gamma I_n, C)$ are stabilizable, detectable, respectively. Any matrix L such that $A - LC$ is Hurwitz is called an observer gain matrix.

Agent models: The dynamics of the non-identical agents are described by linear state-space models. The agent index set is defined as $\mathcal{N} = \{1, \dots, N\}$, where N is the number of agents in the group. The dynamics of the undisturbed agents are described by $\dot{x}_k = A_k x_k + B_k u_k$, where $x_k(t) \in \mathbb{R}^{n_k^x}$ is the state and $u_k(t) \in \mathbb{R}^{n_k^u}$ is the control input of agent $k \in \mathcal{N}$. The cooperative control problem is formulated in terms of the generalized plant

$$\dot{x}_k = A_k x_k + B_k u_k + B_k^{d^g} d^g + B_k^{d^\ell} d_k^\ell \tag{1a}$$

$$y_k = C_k x_k + D_k u_k + D_k^{d^g} d^g + D_k^{d^\ell} d_k^\ell \tag{1b}$$

$$e_k = C_k^e x_k + D_k^e u_k + D_k^{ed^g} d^g + D_k^{ed^\ell} d_k^\ell \tag{1c}$$

where $y_k(t) \in \mathbb{R}^{n_k^y}$ is the measurement output of agent k and $d^g(t) \in \mathbb{R}^{n^{d^g}}$, $d_k^\ell(t) \in \mathbb{R}^{n_k^{d^\ell}}$ are external signals specified next. The regulation error $e_k(t) \in \mathbb{R}^{n_k^e}$ is defined such that asymptotic tracking and disturbance rejection is equivalent to $e_k(t) \to 0$ as $t \to \infty$ for all initial conditions.

External signals: Two types of external input signals affecting the group are considered: a global signal that affects all agents and local signals that affect individual agents in the group. Each of these signals represents a generalized disturbance which may consist of reference signals and disturbances. The global signal $d^g(t)$, $t \geq 0$, is a solution of

$$\dot{d}^g = S^g d^g, \tag{2}$$

called global exosystem, where $\sigma(S^g) \subset \mathbb{C}^0 \cup \mathbb{C}^+$. The local signal $d_k^\ell(t)$, $t \geq 0$, is a solution of the local exosystem system

$$\dot{d}_k^\ell = S_k^\ell d_k^\ell, \tag{3}$$

where $\sigma(S_k^\ell) \subset \mathbb{C}^0 \cup \mathbb{C}^+$, for all $k \in \mathcal{N}$.

Remark 1. The exosystems (2), (3) could be combined into a single large exosystem generating all disturbances acting on the group. In that sense, the present formulation does not enlarge the problem class compared to Su and Huang (2012b,c). The main benefit of taking the structure of the exosystem explicitly into account is the decomposition of the regulator equations and the reduction of the controller dimension, as we will see later in Lemma 1 and Theorem 3, respectively.

Information structure: All agents have communication capabilities. The communication topology is described by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, with node set $\mathcal{N} = \{1, \dots, N\}$ corresponding to the agent index set and edge set $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$. A directed edge $(j, k) \in \mathcal{E}$ corresponds to possible information flow from agent $j \in \mathcal{N}$ to