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Brief paper

Distributed receding horizon control of constrained nonlinear vehicle formations with guaranteed γ-gain stability[☆]

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ABSTRACT

This paper investigates the distributed receding horizon control (RHC) problem of a vehicle platoon with nonlinear dynamics and subject to system constraints, where each vehicle can communicate with its immediate predecessor and follower. A novel optimization problem and detailed distributed RHC algorithm are designed in order to keep a platoon formation, and further to ensure neighbor γ-gain stability (which is a new notion proposed in this paper and generalizes the string stability). The sufficient conditions on ensuring closed-loop stability and neighbor γ-gain stability are established, respectively. Finally, simulation studies are provided to verify the theoretical results. It is shown that it is possible to achieve certain control performance (i.e., γ-gain stability) and keep a formation simultaneously for the nonlinear vehicle platoon using distributed RHC.

1. Introduction

In recent years, the distributed control of vehicle formations has received considerable attention due to its wide applications in automobile industry, multi-robotic systems, and aerospace explorations. In the literature of distributed control, many interesting results have been reported for solving cooperative control problems such as consensus (Li & Yan, 2015; Olfati-Saber & Murray, 2004), formation control (Lin, Francis, & Maggiore, 2005), flocking (Olfati-Saber, 2006) and cooperative stabilization (Dunbar & Murray, 2006; Li & Shi, 2014a). However, in the formation control, most of the results are only focused on vehicle formations with simple linear dynamics. Furthermore, few results pay attention to the control performance during formation procedure that is likely a practical requirement. To advance the research in these two aspects, in this paper, we are interested in the distributed receding horizon control (RHC) problem of a vehicle platoon with *nonlinear dynamics and system constraints*, in which the vehicle platoon needs to be controlled to not only keep a formation, but also achieve certain control performance (i.e., neighbor γ-gain stability).

The related results in the literature are discussed as follows. The distributed RHC problem of unconstrained continuous-time (CT) nonlinear vehicle systems with coupled cost function is investigated in Dunbar and Murray (2006), where the overall stability is established based on the summation of the optimal cost function. The similar problem of discrete-time (DT) nonlinear systems with coupled cost function and constraints is studied in Keviczky, Borrelli, and Balas (2006), where the individual cost function is proved to be qualified as the Lyapunov function for each subsystem and the stability condition has been established accordingly. Furthermore, to handle disturbances, Richards et al. propose the robust distributed RHC strategy for the formation control problem of DT linear agent systems with coupled constraints in Richards and How (2007); the robust RHC problem of CT nonlinear vehicle systems is studied in Li and Shi (2014a), Li and Shi (2014b), where a robustness constraint is introduced to ensure closed-loop robust stability. The distributed RHC problem of unconstrained nonlinear vehicle platoon with "look-ahead" communication fashion is studied in Dunbar and Caveney (2012), where an extra constraint is added to ensure predecessor–follower string stability. In Wang and Ding (2014), the distributed RHC is utilized as a strategy for solving

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the formation control problem of nonholonomic vehicles, where the collision avoidance is addressed by adding coupled constraints.

Since the formation problem may be converted into a consensus problem, the distributed RHC-based consensus problem has also received much attention. For example, in Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009), Ferrari-Trecate et al. develop a distributed RHC strategy for the consensus problem of subsystems modeled by first-order and second-order integrators with constraints, by resorting to the optimal path-based approach, and establish sufficient conditions to ensure consensus. Johansson et al. propose to a RHC-based consensus algorithm for general linear agent systems via a negotiation strategy in Johansson, Speranzon, Johansson, and Johansson (2008), but the consensus condition is not provided. In Li and Yan (2015), the distributed RHC for consensus problem of general linear systems is investigated and a necessary and sufficient condition is presented.

Although much progress has been made in the distributed RHC problem for vehicle platoons, most of them are only focused on the stability issue (such as stability for consensus and formation stability), but ignore the control performance for the vehicle platoon, with few exceptions in Dunbar and Caveney (2012), Wang and Ding (2014). Thus, the distributed RHC for simultaneously ensuring closed-loop stability and achieving control performance is largely scarce. In addition, the system constraint is also a widely ignored practical issue in most of the results on RHC-based formation control.

In this paper, we study the distributed RHC problem of a CT nonlinear vehicle platoon by considering the state and input constraints, where each vehicle can exchange information with its neighbors. We aim at designing a distributed RHC strategy to keep a rigid formation of the vehicle platoon (i.e., closed-loop stability), and achieve γ-gain stability (i.e., control performance) simultaneously. Intuitively speaking, the concept of γ-gain stability is to ensure the maximum changes of each vehicle's and its neighbors' state satisfy certain relation with a given gain γ, and the detailed definitions are given below in Section 2. With the γ-gain stability, each vehicle will change cooperatively with its neighbors during formation procedure. A significantly different initial version has been submitted to Li, Shi, and Yan (submitted for publication). The main contributions of this paper are as follows:

- A novel distributed RHC algorithm is proposed for the nonlinear vehicle platoon with constraints and heterogeneous dynamics, in which a new cost function is developed for keeping vehicle formation, and a gain stability constraint is designed to ensure neighbor γ-gain stability. The stability conditions on how to design the cost function and terminal constraints of the overall vehicle platoon are established.
- Three types of control performance indices (i.e., the predecessor–follower, the follower–predecessor and neighbor γ-gain stability) of the overall vehicle platoon have been proposed. The sufficient conditions under which the vehicle platoon are the predecessor–follower, the follower–predecessor, and neighbor γ-gain stable are established, respectively.

In comparison with the work in Dunbar and Caveney (2012), this paper extends it in two aspects: (1) we generalize the work in Dunbar and Caveney (2012) from the case where subsystems are free of system constraints to a more general case where subsystems are subject to state and input constraints; (2) we extend the concept of predecessor–follower string stability in Dunbar and Caveney (2012) to the concept of neighbor γ-gain stability, design a novel distributed RHC algorithm and completely re-establish sufficient conditions to ensure the neighbor γ-gain stability and show the improvement over the predecessor–follower string stability in the simulation. In addition, we consider bi-directional communication networks instead of the one-directional networks in Dunbar and Caveney (2012).

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. A symmetric and positive definite (positive semi-definite) matrix P is written as $P > 0$ ($P \geqslant 0$). For a given vector $x \in \mathbb{R}^n$ and a matrix $P \geqslant 0$, we use $\|x\|$ and $\|x\|_P = \sqrt{x^{\mathrm{T}}Px}$ to denote its Euclidean norm and P-weight norm. Given a matrix P, its maximum and minimum eigenvalues are denoted by $\bar{\lambda}(P)$ and $\underline{\lambda}(P)$, respectively.

2. Problem statement and modeling

Consider a vehicle platoon consisting of N vehicles moving in a one-dimensional space, in which the leader vehicle is denoted by p_1 and the last follower is denoted by p_N. Given a vehicle p_i, its immediate follower is denoted by p_{i+1}, where $i = 1, \ldots, N-1$, and its immediate leader (i.e., predecessor) is denoted by p_{i-1}, where $i = 2, \ldots, N$. In such a platoon, each vehicle p_i, $i \in \{2, \ldots, N-1\}$, can receive information from its immediate follower p_{i+1} and immediate leader p_i, except the leader and the last follower. The leader vehicle p_1 can only get information from p_2 and the last follower p_N can only receive information from p_{N-1}.

The leader vehicle needs to track a signal $y(t)$ with a constant speed v and all the followers need to follow its immediate leader keeping a distant $d > 0$. The position of each vehicle p_i is denoted by $z_i(t)$, and its velocity is denoted by $\dot{z}_i(t)$. Define the relative position error and relative velocity error as $e_i(t) = z_i(t)+(i-1)d-y(t)$, and $v_i(t) = \dot{z}_i(t)-v$, where $i \in \{1, \ldots, N\}$. Here, it is assumed that each vehicle knows v via initialization and can obtain $z_i(t)$ by absolute position sensors, e.g. GPS. Define $x_i(t) = [e_i(t), v_i(t)]^{\mathrm{T}}$ as the state vector.

For each vehicle p_i, the dynamics can be modeled as

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)), \tag{1}$$

with constraints $x_i(t) \in \mathcal{X}_i$, $u_i(t) \in \mathcal{U}_i$, where $\mathcal{X}_i \subseteq \mathbb{R}^2$ and $\mathcal{U}_i \subseteq \mathbb{R}$ are compact sets and contain the origin as their interior point.

Definition 1 (*Predecessor–follower and Follower–predecessor γ-gain Stability*)**.** For the vehicle platoon followed by subsystems described by (1), given a gain bound $0 < \gamma < \infty$, if there exist constants $\gamma_i \in (0, \gamma]$ such that the closed-loop trajectories satisfy $\sup_{t\geqslant 0} \|x_i(t)\| \leqslant \gamma_i \sup_{t\geqslant 0} \|x_{i-1}(t)\|$, for all $i = 2, \ldots, N$, then the vehicle platoon is said to be predecessor–follower γ-gain stable; if $\sup_{t\geqslant 0} \|x_{i-1}(t)\| \leqslant \gamma_i \sup_{t\geqslant 0} \|x_i(t)\|$, for all $i = 2, \ldots, N$, then the vehicle platoon is said to be follower–predecessor γ-gain stable.

Remark 1. In Definition 1, if $\gamma \in (0, 1)$, then the predecessor–follower γ-gain stability reduces to the predecessor–follower string stability stated in Dunbar and Caveney (2012). However, the follower–predecessor γ-gain stability and the predecessor–follower γ-gain stability cannot be satisfied simultaneously for $\gamma \in (0, 1)$ in the vehicle platoon with bidirectional communication. For example, if there are two vehicles and the predecessor–follower γ-gain stability is satisfied with $\gamma_2 \in (0, 1)$, i.e., $\sup_{t\geqslant 0} \|x_2(t)\| \leqslant \gamma_2 \sup_{t\geqslant 0} \|x_1(t)\|$, then one has $\sup_{t\geqslant 0} \|x_1(t)\| \geqslant \frac{1}{\gamma_2} \sup_{t\geqslant 0} \|x_2(t)\| > \sup_{t\geqslant 0} \|x_2(t)\|$, implying that γ for the follower–predecessor γ-gain stability is greater than 1. As a result, we generalize the definition of predecessor–follower string stability in Dunbar and Caveney (2012) to Definition 1 without requiring $\gamma \in (0, 1)$.

Definition 2 (*Neighbor γ-gain Stability*)**.** For the vehicle platoon with subsystems described by (1), given a gain bound $0 < \gamma < \infty$, if there exist constants $\gamma_i \in (0, \gamma]$, such that the closed-loop trajectories satisfy $\max\{\sup_{t\geqslant 0} \|x_{i+1}(t)\|, \sup_{t\geqslant 0} \|x_{i-1}(t)\|\} \leqslant \gamma_i \sup_{t\geqslant 0} \|x_i(t)\|$, for all $i = 1, \ldots, N$, where $x_i(t) = 0$ if $i < 0$ or $i > N$, then the vehicle platoon is said to be neighbor γ-gain stable.