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Brief paper

Euler's discretization effect on a twisting algorithm based sliding mode control☆

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ABSTRACT

In this paper, the Euler's discretization of the second order sliding mode control system with the twisting algorithm is studied. It shows that, for all values of discretization steps, initial conditions and allowable parameter setting, the system trajectories are always bounded. It also shows that for the uncertain systems, under a mild condition, the system trajectories are also bounded. Further, the periodic behaviors and limit cycles of the system trajectories are explored with conditions for the existence of periodic orbits formulated. Extensive simulation examples are given to show typical trajectories, and comparison between the first order and second order sliding mode control systems.

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1. Introduction

The second order sliding mode (SOSM) algorithms have been shown to be a very effective tool for chattering attenuation and accuracy improvement (Fridman, 2012; Levant, 1993; Shustin, Fridman, Fridman, & Castaños, 2008). The twisting algorithm based sliding mode control (SMC) system (for short, we call it the twisting system) is one of the simplest and most popular algorithms belonging to the considered class of SOSM. The twisting system and its global finite-time stability were introduced and proven in Levant (1993) for the first time. Since then the twisting system has been widely studied and applied (Polyakov & Poznyak, 2009; Shtessel, Edwards, Fridman, & Levant, 2013). Since the fast developments of digital microprocessor based control technology, extensive research has been done in the last few decades on the discrete-time SOSM systems. The accuracies of higher-order sliding modes under the one step Euler integration method with variable sampling steps were calculated in Levant (2011). A new discrete time super-twisting-like SOSM algorithm was proposed in Salgado, Kamal, Chairez, Bandyopadhyay, and Fridman (2011). However, there has been little work on study of discretization effect on the continuous time SOSM systems apart from the early work in Yan, Yu, and Sun (2014), which showed that Euler's discretization of the twisting system can lead to irregular periodic behaviors. The open questions that need to be addressed are the stability conditions, sensitivity to initial conditions, existence of periodicity in system orbits, etc.

In this paper, we will first study the stability issue of SOSM under Euler's discretization. We will show that for all values of discretization steps and initial conditions the system orbits are always bounded. We will also show that under a mild condition, the trajectories of the uncertain system are also bounded. Furthermore, we will demonstrate that, similar to the results in the discretized equivalent-control based SMC systems (Galias & Yu, 2007; Yu, Wang, & Li, 2012) and the SOSM systems with time delay (Levaggi & Punta, 2006; Shustin, Fridman, & Fridman, 2003), the discretization of the SOSM systems can also induce periodic orbits and limit cycles around the system equilibrium.

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This paper is organized as follows. Section 2 presents the problem statement. In Section 3, bounds of the steady states under both the certain and uncertain systems are derived, and periodicity conditions are explored. In Section 4, simulation results are shown to demonstrate the effectiveness of theoretical results, and some comparison between the first order and second order SMC systems is given.

2. Twisting algorithm based sliding mode control strategy

Consider the twisting system of the form

$$\dot{x}_1 = x_2, \tag{1a}$$

$$\dot{x}_2 = -M_1 \mathrm{sgn}(x_1) - M_2 \mathrm{sgn}(x_2), \tag{1b}$$

where $x \in \mathbb{R}^2$ is a state vector, $M_1 > M_2 > 0$, $\mathrm{sgn}(y) = +1$ for $y \geq 0$, and $\mathrm{sgn}(y) = -1$ for $y < 0$. The system (1) is homogeneous (invariant) with respect to the transformation $(t, x_1, x_2) \mapsto (kt, k^2 x_1, kx_2)$ and that is the so-called 2-sliding homogeneity (Levant, 2011). Since the structure of the plant (1) is intentionally changed by the signum functions of both x_1 and x_2, the twisting system cannot have a monotonic behavior. The trajectory switches into a new parabolic motion every time it intersects the switching lines $x_1 = 0$ and $x_2 = 0$. As a result, the trajectory spirally converges to the origin and the SOSM emerges if a trajectory reaches the origin.

The uncertain twisting system is defined as

$$\dot{x}_1 = x_2, \tag{2a}$$

$$\dot{x}_2 = -g(t,x)(M_1 \mathrm{sgn}(x_1) + M_2 \mathrm{sgn}(x_2)) + f(t,x), \tag{2b}$$

where the smooth functions f, g are unknown and satisfy the conditions $0 < K_m \leq g \leq K_M$, $|f| \leq C$. The reaching time estimate of (2) was found in Polyakov and Poznyak (2009). The uncertain system can be presented as a differential inclusion $\dot{x}_1 = x_2$, $\dot{x}_2 \in -[K_m, K_M](M_1 \mathrm{sgn}(x_1) + M_2 \mathrm{sgn}(x_2)) + [-C, C]$.

In the next section, we first analyze discretization effects on (1) as it is the fundamental form of SOSM, and then we study discretization effects on the uncertain system (2).

3. Analysis of the discretized system

The Euler's discretization of (1) with the time step $h > 0$ generates the following discrete-time system:

$$x_1^{(k+1)} = x_1^{(k)} + h x_2^{(k)}, \tag{3a}$$

$$x_2^{(k+1)} = x_2^{(k)} - hM_1 s_1^{(k)} - hM_2 s_2^{(k)} \tag{3b}$$

where $s_i^{(k)} = \mathrm{sgn}(x_i^{(k)})$ for $i = 1, 2$. Let us denote $\alpha = M_1 + M_2$, $\beta = M_1 - M_2 > 0$ and $\gamma = \alpha/\beta > 1$.

According to the signs of x_1 and x_2, we divide the phase plane into four regions: $Q_0 = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\}$, $Q_1 = \{(x_1, x_2) : x_1 \geq 0, x_2 < 0\}$, $Q_2 = \{(x_1, x_2) : x_1 < 0, x_2 < 0\}$, and $Q_3 = \{(x_1, x_2) : x_1 < 0, x_2 \geq 0\}$. Observe that the regions Q_k are not symmetric with respect to the origin due to the fact that signum functions are asymmetric.

To get a better understanding of a relationship between dynamical behaviors and system parameters let us redefine the system so that it depends on a single parameter. By using the property of 2-sliding homogeneity (Levant, 2011) we introduce the linear change of variables $z_1 = x_1/(h^2\beta)$, $z_2 = x_2/(h\beta)$. In these variables the dynamical system (3) has the form:

$$z_1^{(k+1)} = z_1^{(k)} + z_2^{(k)}, \qquad z_2^{(k+1)} = z_2^{(k)} - g(z^{(k)}), \tag{4}$$

where $g(z) = \delta_j$ for $z \in Q_j$, with $\delta_0 = \gamma$, $\delta_1 = 1$, $\delta_2 = -\gamma$, and $\delta_3 = -1$.

The following lemma states that trajectories of (4) turn around the origin in a clockwise direction visiting region $Q_{(p+1) \bmod 4}$ immediately after leaving region Q_p.

Lemma 1. *If $z^{(i)} \in Q_p$ then there exists k such that $z^{(i+k)} \in Q_{(p+1) \bmod 4}$ and $z^{(i+j)} \in Q_p$ for $0 \leq j < k$. Moreover, in each region Q_p trajectories are monotonic in both z_1 and z_2.*

Proof. From (4) it follows that as long as $z \in Q_0$ the variable z_1 does not decrease and z_2 decreases by γ. It is clear that after a finite number of iterations the trajectory started at $z^{(i)} \in Q_0$ leaves Q_0 and enters Q_1. Similarly, for any initial point $z \in Q_1$, formulae (4) indicate that both z_1 and z_2 decrease monotonically. The trajectory leaves Q_1 and enters Q_2 after a finite number of iterations. Proof for other cases is analogous. □

3.1. Bounds for steady states; stability analysis

In this section, we fully explore the stability issue, which is the most important aspect to study in control systems. First, we formulate three technical lemmas which are necessary to prove the main stability result which is formulated as Theorem 1. Let us assume that for the iterations $z^{(-1)}$ and $z^{(0)}$ there is a switching in the variable z_1, i.e. $\mathrm{sgn}(z_1^{(-1)}) \neq \mathrm{sgn}(z_1^{(0)})$. From Lemma 1 it follows that there are two possibilities: (a) $z^{(-1)} \in Q_3$, $z^{(0)} \in Q_0$, and (b) $z^{(-1)} \in Q_1$, $z^{(0)} \in Q_2$.

Let us define $P(z^{(0)}) = z^{(n)}$, where n is such that there is a switching in the variable z_1 from $z^{(n-1)}$ to $z^{(n)}$ and there is no such switching for $0 \leq k < n$. The map P is a discrete version of the return map defined for the continuous system by the line $z_1 = 0$. However, note that the map P is two-dimensional whereas the return map for a continuous time system is of dimension 1.

Below, we derive a formula for P and we show that if $z_2^{(0)}$ is sufficiently large then $|z_2^{(n)}| < |z_2^{(0)}|$. We restrict formulation and proof for the case $z^{(0)} \in Q_0$. For the case $z^{(0)} \in Q_2$ results are symmetric and proofs are analogous.

Lemma 2. *Let us assume that $z^{(-1)} \in Q_3$ and $z^{(0)} \in Q_0$. Let n be the smallest positive integer such that $z^{(n-1)} \in Q_1$ and $z^{(n)} \in Q_2$, i.e. $z^{(n)} = P(z^{(0)})$. Then*

$$z_1^{(n)} = z_1^{(0)} + n_0 z_2^{(0)} - 0.5\gamma n_0(n_0 - 1) + n_1(z_2^{(0)} - \gamma n_0) - 0.5 n_1(n_1 - 1), \tag{5a}$$

$$z_2^{(n)} = z_2^{(0)} - \gamma n_0 - n_1, \tag{5b}$$

where n_0 and n_1 are the smallest integer numbers satisfying conditions: $n_0 > \gamma^{-1} z_2^{(0)}$, $n_1 > -b + \sqrt{b^2 - 2c}$ with $b = \gamma n_0 - 0.5 - z_2^{(0)}$, $c = 0.5\gamma n_0(n_0 - 1) - z_1^{(0)} - n_0 z_2^{(0)}$ and $n = n_0 + n_1$.

Proof. Let us denote by n_0 the number of iterations the trajectory initiated in $z^{(0)}$ stays in Q_0. From (4), the formula for the n_0th iteration reads

$$z_1^{(n_0)} = z_1^{(0)} + n_0 z_2^{(0)} - 0.5\gamma n_0(n_0 - 1), \tag{6}$$

$$z_2^{(n_0)} = z_2^{(0)} - n_0\gamma. \tag{7}$$

Since $z^{(n_0-1)} \in Q_0$ and $z^{(n_0)} \in Q_1$, we have $z_2^{(n_0-1)} = z_2^{(0)} - (n_0 - 1)\gamma \geq 0$ and $z_2^{(n_0)} = z_2^{(0)} - n_0\gamma < 0$ and hence n_0 satisfies the condition

$$n_0 \in \left(\gamma^{-1} z_2^{(0)}, \gamma^{-1} z_2^{(0)} + 1\right]. \tag{8}$$

Let n_1 be the number of iterations the trajectory initiated in $z^{(n_0)}$ stays in Q_1. It follows that $z_1^{(n_0+n_1)} = z_1^{(n_0)} + n_1 z_2^{(n_0)} - 0.5 n_1(n_1 - 1)$, $z_2^{(n_0+n_1)} = z_2^{(n_0)} - n_1$. Since $z_1^{(n_0+n_1)} < 0$ and $z_1^{(n_0+n_1-1)} \geq 0$ it follows that n_1 is the smallest positive integer such that $z_1^{(n_0)} + n_1 z_2^{(n_0)} - 0.5 n_1(n_1 - 1) < 0$. Substituting $z_{1,2}^{(n_0)}$

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