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Consensus networks with switching topology and time-delays over finite fields ☆

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ABSTRACT

The consensus problem in networks with both switching topology and time-delays over finite fields is investigated in this paper. The finite field, which is a kind of finite alphabet, is considered due to the fact that networks often possess limited computation, memory, and capabilities of communication. First, by graph-theoretic method, one necessary and sufficient condition is derived for finite-field consensus of switching networks without time-delays. Subsequently, another necessary and sufficient condition on finite-field consensus without time-delays is provided based on FFC property of matrices associated with switching networks. Moreover, by means of the results on delay-free networks, some necessary and sufficient conditions for finite-field consensus of networks with both switching topology and time-delays are obtained. Additionally, it can be shown that switching networks with time-delays present in each self-transmission cannot achieve consensus.

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1. Introduction

A multi-agent system or network consists of a group of agents or nodes that communicate with each other locally, aiming to achieve some goals by designing control strategies, which has attracted an increasing interest from numerous researchers over the past decade. Due to a large volume of potential applications in many areas, multi-agent systems have a number of distinct research directions including consensus (Hu, Lam, & Liang, 2013; Olfati-Saber & Murray, 2004; Su, Chen, Lam, & Lin, 2013; Su, Chen, Wang, & Lam, 2014; Su, Chen, Wang, & Lin, 2011), flocking (Su, Wang, & Lin, 2009a; Tanner, Jadbabaie, & Pappas, 2007), formation control (Meng, Jia, Du, & Zhang, 2014), etc. As an important distributed feature of networks, consensus aims at for all agents reaching an agreement of interest decided by themselves, which has been substantially investigated due to a large number of applications in a range of domains, such as spacecrafts and robotics (Ren, Beard, & Atkins, 2007). Up to now, consensus has been deeply studied for various network models, including time-independent networks, networks with broad and synchronous communication, networks with linear and nonlinear dynamics, networks with infinite communication bandwidth, networks with time-varying topologies, networks with gossip and asynchronous communication, and networks with link failures and so on (Chen, Wang, & Li, 2012; Fagiolini & Bicchi, 2013; Hadjicostis & Charalambous, 2014; Li, Liu, Wang, & Yin, 2014; Lin & Jia, 2009; Moreau, 2005; Ni & Cheng, 2010; Nuño, Ortega, Basanez, & Hill, 2011; Proskurnikov, 2013; Song, Cao, & Liu, 2010; Su, Wang, & Lin, 2009b; Tahbaz-Salehi & Jadbabaie, 2008; Wang & Xiao, 2007; You, Li, & Xie, 2013; Zhang & Tian, 2009; Zhao, Hill, & Liu, 2012; Zhou & Lin, 2014).

Recently, instead of real numbers, finite fields have been taken into consideration for the consensus problem in networks for the sake of safety and memory constraints, etc. (Pasqualetti, Borra, & Bullo, 2014; Sundaram & Hadjicostis, 2013; Xu & Hong, 2014). For instance, some necessary and sufficient conditions on finite-field consensus of fixed networks with discrete-time iteration were developed completely in Pasqualetti et al. (2014), in which the authors also provided its applications to average consensus and pose estimation in sensor networks. However, in the context

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of finite fields, there are few research reports focusing on the consensus problem for networks with switching topology and time-delays, which inspire this paper.

This paper addresses the consensus problem for networks with switching topology and time-delays over finite fields. The finite field, as a kind of finite alphabet, is taken into account since networks often undertake limited computation, memory, and capabilities of communication. Generally speaking, this paper extends the work in Pasqualetti et al. (2014) to the case with switching topology and time-delays. Provided that switching topology and delays complicate the structure of a network which further leads to the complexity of the union of transition graphs of adjacency matrices, more careful observations on the structure of the union of transition graphs are needed to derive one necessary and sufficient condition for consensus with switching topology by graph-theoretic method. In addition, intrinsically distinct from Pasqualetti et al. (2014), FFC property of finite product of adjacency matrices is raised here to obtain another necessary and sufficient condition for consensus with switching topology. Furthermore, regarding networks with both switching topology and time-delays, several necessary and sufficient conditions are presented to guarantee finite-field consensus using the results on delay-free networks. Moreover, it is shown that switching networks with time-delays present in each self-transmission cannot reach consensus.

2. Preliminaries

A finite field $\mathbb{F}$ is a finite set of elements with addition and multiplication operations satisfying the following axioms (Lidl, 1997; Pasqualetti et al., 2014; Sundaram & Hadjicostis, 2013; Xu & Hong, 2014):

- Closure under addition and multiplication. $a+b \in \mathbb{F}$, $a \cdot b \in \mathbb{F}$, $\forall a, b \in \mathbb{F}$;
- Associativity of addition and multiplication. $a+(b+c) = (a+b)+c$, $a\cdot(b\cdot c) = (a\cdot b)\cdot c$, $\forall a, b, c \in \mathbb{F}$;
- Commutativity of addition and multiplication. $a+b = b+a$, $a \cdot b = b \cdot a$, $\forall a, b \in \mathbb{F}$;
- Existence of additive and multiplicative identity elements. $\forall a \in \mathbb{F}$, $\exists 0, 1$, such that $a+0 = a$, $a \cdot 1 = a$;
- Existence of additive and multiplicative inverse elements. $\forall a \in \mathbb{F}$, $\exists b, c \in \mathbb{F}$, such that $a+b = 0$, $a \cdot c = 1$ $(a \neq 0)$;
- Distributivity of multiplication over addition. $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$, $\forall a, b, c \in \mathbb{F}$.

Denote by $\mathcal{G}_N = (\mathcal{V}_N, \mathcal{E}_N)$ a graph with N nodes, a set of vertices $\mathcal{V}_N = \{v_1, v_2, \ldots, v_N\}$ and a set of edges $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$. $v_i \to v_j$ (or $(v_i, v_j) \in \mathcal{E}_N$) denotes an edge in which node i sends information to node j. A graph is called undirected if $(v_i, v_j) \in \mathcal{E}_N$ implies $(v_j, v_i) \in \mathcal{E}_N$, and directed otherwise. For a node i, the in-degree and out-degree of $v_i \in \mathcal{V}_N$ equal the numbers of the in-neighbor set $\mathcal{N}_i^+ = \{v_j \in \mathcal{V}_N : (v_j, v_i) \in \mathcal{E}_N\}$ and the out-neighbor set $\mathcal{N}_i^- = \{v_j \in \mathcal{V}_N : (v_i, v_j) \in \mathcal{E}_N\}$, respectively. The adjacency matrix $A_N = (a_{ij}) \in \mathbb{F}_p^{N\times N}$ is defined as: $a_{ij} > 0$ if $v_j \in \mathcal{N}_i^+$, $a_{ij} = 0$ $(i \neq j)$ otherwise, and self-loops are allowed. A directed path in a directed graph is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots$. A cycle is a path that shares the same first and last vertex. A directed tree is a directed graph, where every node has exactly one parent except one node called root. A root (resp. globally reachable node) is a node which has a directed path to (resp. from) every node in the graph, including itself. The length of a path (resp. cycle) equals the number of edges in the path (resp. cycle). A directed graph is called strongly (resp. weakly) connected if there is a directed (resp. undirected) path between any two nodes. Two subgraphs of the same graph are called disjoint if they have no common nodes.

3. Problem statement

Consider a switching network with N nodes over a finite field. For simplicity, throughout this paper the prime field $\mathbb{F}_p := \{0, 1, 2, \ldots, p-1\}$ is considered, where p is a prime number. However, the results here can be easily extended to general finite fields. The network has the dynamics

$$x_i(t+1) = \sum_{j\in\mathcal{N}_i^+(t)\cup\{i\}} a_{ij}^{s(t)} x_j(t-\tau_{ij}), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{F}_p$ is the state of this network, and τ_{ij}'s are time-delays experienced by information transmission on the link from node j to node i satisfying $0 \leq \tau_{ij} \leq \tau$ for some constant $\tau > 0$, that is, τ_{ij}'s are bounded, $i, j = 1, 2, \ldots, N$. Meanwhile, the adjacency matrix $A_{s(t)} = (a_{ij}^{s(t)}) \in \mathbb{F}_p^{N\times N}$ is row-stochastic (see Definition 2) and $s(t) : \{0, 1, 2, \ldots\} \to \mathcal{S}$ is the switching signal with finite index set $\mathcal{S} := \{1, 2, \ldots, \nu\}$. Therefore, for any $k \in \mathcal{S}$, when $s(t) = k$, the subnetwork (1) with $A_{s(t)} = A_k$ is activated. As a delayed network, it is assumed that subnetwork $s(t)$ takes the initial states $x(t-1), x(t-2), \ldots, x(t-\tau)$ when subnetwork $s(t-1)$ is switched to subnetwork $s(t)$ at time t, where $x(t) := (x_1(t), x_2(t), \ldots, x_N(t))^T \in \mathbb{F}_p^N$. Note that the addition and multiplication operations in the network (1) are performed by modulo, that is, taking the remainder after divided by p.

For some $k \in \mathcal{S}$, the transition graph of corresponding fixed network $x(t+1) = A_k x(t)$ is defined as $\mathcal{G}_{A_k}^* = (\mathcal{V}_{A_k}^*, \mathcal{E}_{A_k}^*)$ with vertex set $\mathcal{V}_{A_k}^* = \{v : v \in \mathbb{F}_p^N\}$ and edge set $\mathcal{E}_{A_k}^* = \{(v_i, v_j) : v_j = A_k v_i,\ v_i, v_j \in \mathbb{F}_p^N\}$. This transition graph comprises of disjoint weakly-connected subgraphs, and only one cycle, maybe of unit length, is contained in each subgraph which embraces a globally reachable node (Hernández Toledo, 2005; Pasqualetti et al., 2014). As for switching network (1), denote by $\mathcal{G}_A^* = (\mathcal{V}_A^*, \mathcal{E}_A^*)$ the union graph of $\mathcal{G}_{A_k}^*$ for all $k \in \mathcal{S}$, that is, $\mathcal{V}_A^* = \mathcal{V}_{A_k}^* = \{v : v \in \mathbb{F}_p^N\}$ and edge set $\mathcal{E}_A^* = \bigcup_{k\in\mathcal{S}} \mathcal{E}_{A_k}^*$. To proceed, the notion of consensus of networks over finite fields is defined as follows.

Definition 1. The network (1) over $\mathbb{F}_p$ can achieve (finite-time) consensus if for any initial states in $\mathbb{F}_p^N$ and any switching signal $s(t)$, there exist a finite time $T \in \mathbb{N}$ and some constant $\eta \in \mathbb{F}_p$ such that $x(T+k) = x(T) = \eta\mathbf{1}$ for all $k \in \mathbb{N}$.

Note that consensus of networks over finite fields can be always achieved in finite time since there are only finite states in networks over finite fields. For brevity, two concepts are introduced as follows.

Definition 2. Over the finite field $\mathbb{F}_p$, (1) a matrix $M \in \mathbb{F}_p^{n\times n}$ is called row-stochastic if each row sums to 1; (2) a row-stochastic matrix $M \in \mathbb{F}_p^{n\times n}$ is called finite-field consensusable (FFC, for short) if M has a simple eigenvalue 1 and all other eigenvalues 0, that is, its characteristic polynomial is $P_M(\lambda) = \lambda^{n-1}(\lambda - 1)$.

This definition is reasonable due to the following result that states two necessary and sufficient conditions for finite-field consensus, which are conducive to consensus analysis later.

Theorem 1 (*Pasqualetti et al., 2014*)**.** *For a fixed network $x(t+1) = Mx(t)$ over $\mathbb{F}_p$, the following statements are equivalent:* (1) *this network can achieve consensus;* (2) *the transition graph of M contains exactly p cycles and all of them are unit cycles around the vertices $\eta\mathbf{1}$ for $\eta \in \mathbb{F}_p$;* (3) *the matrix M is FFC.*

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