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Zeros of networked systems with time-invariant interconnections ☆

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ABSTRACT

This paper studies zeros of networked linear systems with time-invariant interconnection topology. While the characterization of zeros is given for both heterogeneous and homogeneous networks, homogeneous networks are explored in greater detail. In the current paper, for homogeneous networks with time-invariant interconnection dynamics, it is illustrated how the zeros of each individual agent's system description and zeros definable from the interconnection dynamics contribute to generating zeros of the whole network. We also demonstrate how zeros of networked systems and those of their associated blocked versions are related.

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1. Introduction

Recent developments of enabling technologies such as communication systems, cheap computation equipment and sensor platforms have given great impetus to the creation of networked systems. Thus, this area has attracted significant attention worldwide and researchers have studied networked systems from different perspectives (see e.g. Olfati-Saber & Murray, 2002, Sinopoli, Sharp, Schenato, Schaffert, & Sastry, 2003, Tanner, Jadbabaie, & Pappas, 2003 and Zamani & Lin, 2009). In particular, in view of the recent chain of events (Falliere, Murchu, & Chien, 2011; Gorman, 2009; Rid, 2012), the issues of security and cyber threats to the networked systems have gained growing attention. This paper uses system theoretic approaches to deal with problems involved with the security of networks.

Recent works have shown that control theory can be used as an effective tool to detect and mitigate the effects of cyber attacks on the networked systems; see for example Amin, Cárdenas, and Sastry (2009), Cárdenas et al. (2011), Gupta, Langbort, and Basar (2010), Mo et al. (2012), Sridhar, Hahn, and Govindarasu (2012), Teixeira, Shames, Sandberg, and Johansson (2012) and the references listed therein. The authors of Teixeira et al. (2012) have introduced the concept of *zero-dynamics attacks* and shown how attackers can use knowledge of networks' zeros to produce control commands such that they are not detected as security threats.[1] They have further shown that zeros of networks provide valuable information relevant to detecting cyber attacks. The authors in Teixeira et al. (2012) were more concerned with mitigating such attacks and did not provide a detailed discussion about zeros of the networked systems. In addition to this, even though various aspects of the dynamics of networked systems have been extensively studied in the literature, see e.g. Fax and Murray (2004), Olfati-Saber, Fax, and Murray (2007) and Ren, Beard, and Atkins (2007), to the authors' best knowledge, the zeros of networked systems have not been studied in any detail except in Zamani, Helmke, and Anderson (2013). The current paper establishes a link between the problem of zero-dynamics attacks and the analysis of zeros that has been recorded in Zamani et al. (2013). Furthermore, several new results are introduced in the current paper compared to its preliminary conference version including the provision of proofs of certain results which were not part of the conference version.

This paper examines the zeros of networked systems in more depth. Our focus is on networks of finite-dimensional linear discrete-time dynamical systems that arise through static

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[1] This is discussed further in the next section.

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interconnections of a finite number of such systems. Such models arise naturally in applications of linear networked systems, e.g. for cyclic pursuit (Marshall, Broucke, & Francis, 2004); shortening flows in image processing (Bruckstein, Sapiro, & Shaked, 1995), or for the discretization of partial differential equations (Brockett & Willems, 1974).

Our ultimate goal is to analyze the zeros of networked systems with periodic, or more generally time-varying interconnection topology. An important tool for this analysis is blocking or lifting technique for networks with time-invariant interconnections. Note that blocking of linear time-invariant systems is useful if not standard in design of controllers for linear periodic systems as shown by Chen and Francis (1995) and Khargonekar, Poolla, and Tannenbaum (1985). The authors of Bolzern, Colaneri, and Scattolini (1986) and Grasselli and Longhi (1988) have examined zeros of blocked systems obtained from blocking of time-invariant systems. Their works have been extended in Chen, Anderson, Deistler, and Filler (2012) and Zamani, Chen, Anderson, Deistler, and Filler (2011). However, these earlier contributions do not take any underlying network structure into consideration. In this paper, we introduce some results that provide a first step in that direction.

It is worthwhile noting the blocking technique has been used in the networked systems literature for both control and identification purposes. For instance, the authors in Haber and Verhaegen (2014) have exploited this technique to identify the system parameters in a networked system via the subspace approach. The same set of authors have employed the blocking technique to study moving horizon estimation problem for networked systems (Haber & Verhaegen, 2013). In Montestruque and Antsaklis (2006) the authors have utilized the blocking technique to provide a sufficient and necessary condition for stability of a class of networked systems with communication bandwidth limitation. A similar problem has been addressed in Garcia and Antsaklis (2010) using the blocking.

The structure of this paper is as follows. First, in Section 2 we introduce state-space and higher order polynomial system models for time-invariant networks of linear systems. A central result used is the strict system equivalence between these different system representations. Moreover, we completely characterize both finite and infinite zeros of arbitrary heterogeneous networks. For homogeneous networks of identical SISO systems more explicit results are provided in Section 3. Homogeneous networks with a circulant coupling topology are studied as well. In Section 4, a relation between the transfer function of the blocked system and the transfer function of the associated unblocked system is explained. We then relate the zeros of blocked networked systems to those of the corresponding unblocked systems, generalizing work in Chen et al. (2012), Zamani, Anderson, Helmke, and Chen (2013) and Zamani et al. (2011). Finally, Section 5 provides the concluding remarks.

2. Problem statement and preliminaries

We consider networks of N linear systems, coupled through constant interconnection parameters. Each agent is assumed to have the state-space representation as a linear discrete-time system

$$\begin{aligned} x_i(t+1) &= A_i x_i(t) + B_i v_i(t) \\ w_i(t) &= C_i x_i(t), \quad i = 1, \dots, N. \end{aligned} \tag{1}$$

Here, $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m_i}$ and $C_i \in \mathbb{R}^{p_i \times n_i}$ are the associated system matrices. We assume that each system is reachable and observable and that the agents are interconnected by static coupling laws

$$v_i(t) = \sum_{j=1}^{N} L_{ij} w_j(t) + R_i u(t) \in \mathbb{R}^{m_i}, \tag{2}$$

with $L_{ij} \in \mathbb{R}^{m_i \times p_j}$, $R_i \in \mathbb{R}^{m_i \times m}$ and $u(t) \in \mathbb{R}^m$ denoting an external input applied to the whole network. Further, we assume that there is a p-dimensional interconnected output given by

$$y(t) = \sum_{i=1}^{N} S_i w_i(t) + Du(t), \quad \text{with } S_i \in \mathbb{R}^{p \times p_i}, \ i = 1, \dots, N. \tag{3}$$

Define $\overline{m} = \sum_{i=1}^N m_i$, $\overline{p} = \sum_{i=1}^N p_i$, $\overline{n} = \sum_{i=1}^N n_i$ and *coupling matrices*

$$L = (L_{ij})_{ij} \in \mathbb{R}^{\overline{m} \times \overline{p}}, \qquad R = \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix} \in \mathbb{R}^{\overline{m} \times m},$$

$$S = (S_1, \dots, S_N) \in \mathbb{R}^{p \times \overline{p}}, \qquad D \in \mathbb{R}^{p \times m},$$

as well as *node matrices*

$$\begin{aligned} &A = \operatorname{diag}(A_1, \dots, A_N), \qquad B = \operatorname{diag}(B_1, \dots, B_N), \\ &C = \operatorname{diag}(C_1, \dots, C_N), \qquad x(t) := \begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix} \in \mathbb{R}^{\overline{n}}. \end{aligned} \tag{4}$$

Then the closed-loop system is

$$\begin{aligned} x(t+1) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) + Du(t), \end{aligned} \tag{5}$$

with matrices

$$\mathbf{A} := A + BLC, \qquad \mathbf{B} := BR, \qquad \mathbf{C} := SC. \tag{6}$$

One could also start by assuming that each system (1) is defined in terms of a restricted version of Rosenbrock-type equations (Rosenbrock, 1970) i.e. by systems of higher order difference equations

$$\begin{aligned} T_i(\sigma)\xi_i(t) &= U_i(\sigma)v_i(t) \\ w_i(t) &= V_i(\sigma)\xi_i(t). \end{aligned} \tag{7}$$

Here σ denotes the shift operator that acts on sequences of vectors $(\xi(t))_t$ as $(\sigma\xi(t)) = \xi(t+1)$. Furthermore, T_i, U_i, V_i denote polynomial matrices of sizes $T_i(z) \in \mathbb{R}[z]^{r_i \times r_i}$, $U_i(z) \in \mathbb{R}[z]^{r_i \times m_i}$ and $V_i(z) \in \mathbb{R}[z]^{p_i \times r_i}$, respectively. We always assume that $T_i(z)$ is *nonsingular*, i.e. that $\det T_i(z)$ is not the zero polynomial. Moreover, the system (7) is assumed to be strictly proper, i.e. we assume that the associated transfer function

$$G_i(z) = V_i(z)T_i(z)^{-1}U_i(z) \tag{8}$$

is strictly proper. Following Fuhrmann (1977), any strictly proper system of higher order difference equations has an associated state-space realization (A, B, C), the so-called **shift realization**, such that the polynomial matrices

$$\begin{pmatrix} zI - A & -B \\ C & 0 \end{pmatrix}, \qquad \begin{pmatrix} T(z) & -U(z) \\ V(z) & 0 \end{pmatrix} \tag{9}$$

are strict system equivalent (Fuhrmann, 1977). If the first order representation (1) is strict system equivalent to the higher order system (7) then of course the associated transfer functions coincide, i.e. we have

$$C_i(zI - A_i)^{-1}B_i = V_i(z)T_i(z)^{-1}U_i(z). \tag{10}$$

Throughout this paper we assume that the first order and higher order representations i.e. the systems (1) and (7), are chosen to be of minimal order, respectively. This is equivalent to the controllability and observability of the shift realizations (1) associated with these representations (7). It is also equivalent to the simultaneous left coprimeness of $T_i(z), U_i(z)$ and the right

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