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Formation control of a multi-agent system subject to Coulomb friction ☆

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ABSTRACT

This paper considers the formation control problem for a network of point masses which are subject to Coulomb friction. A dynamical model including the planar discontinuous friction force is presented in the port-Hamiltonian framework. Moreover, continuous and discontinuous controllers are designed in order to achieve a desired prescribed formation. The main results are derived using tools from nonsmooth Lyapunov analysis. It is shown that the continuous static feedback controller fails to achieve the exact formation, while the discontinuous controller achieves the desired task exactly. Numerical simulations are provided to illustrate the effectiveness of the approach.

1. Introduction

Cooperative motion coordination of mobile agents has attracted increasing attention in recent years owing to its wide range of applications from biology and social networks to sensor/robotic networks. Distributed formation control is a cooperative control problem which aims at achieving a desired collective behavior, mainly forming a desired geometrical shape, for a group of agents using local feedback laws rather than a centralized controller (Arcak, 2007).

The dynamics of agents play an important role in the problem of formation control. In this regard, different classes of dynamic agents have been considered e.g. Bai, Arcak, and Wen (2011), Nuño, Ortega, Basañez, and Hill (2011), Ren (2009) and van der Schaft and Maschke (2013). For dynamic agents, the dissipation due to friction forces plays a key role in the stability analysis of the whole network e.g. Bai et al. (2011), Jafarian and De Persis (2015) and Vos, Scherpen, and van der Schaft (2012). In the literature, to the authors best knowledge, only continuous friction forces have been considered for the formation control problem of networks. This paper considers formation control of a group of agents in the presence of Coulomb friction which is a discontinuous friction law (van de Wouw & Leine, 2004). Coulomb friction is a quantification of the friction force that exists between two (dry) surfaces in contact with each other. Coulomb friction renders the networked system nonsmooth, thereby requiring tools from nonsmooth systems for the analysis. Nonsmooth stability theory has been already considered in formation keeping control, e.g. in finite-time consensus algorithms (Cortés, 2006), and in quantized coordination (Ceragioli, De Persis, & Frasca, 2011; De Persis & Jayawardhana, 2012; Jafarian & De Persis, 2013, 2015).

In this paper, the model of the network and the formation controller design are defined in the port-Hamiltonian framework (Duindam, Macchelli, & Stramigioli, 2009; van der Schaft & Jeltsema, 2014). Being an energy-based modeling framework (Ortega, van der Schaft, Mareels, & Maschke, 2002), the port-Hamiltonian framework enables the modeling of a physical phenomenon like Coulomb friction in a natural way (i.e., the model provides a clear physical interpretation). The port-Hamiltonian framework interconnects the various sub-systems in a power preserving manner using the so-called *power ports* (Duindam et al., 2009). Moreover,

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the Hamiltonian equals the total energy stored in the system and can be used as a Lyapunov function. In addition, one can design controllers using energy-related elements, e.g. springs, in order to shape the total energy of the system into a desired one. The term *virtual spring* used throughout this paper refers to the application of this concept in the control design. We build upon the recently introduced theory of *port-Hamiltonian systems on graphs* (van der Schaft & Maschke, 2013) where graph theory is used to model the interconnection amongst agents in the network, and upon nonsmooth formation control (Jafarian & De Persis, 2013, 2015).

We consider a network of point masses moving in $\mathbb{R}^2$ and assume that each of the agents is subject to Coulomb friction. To achieve the desired formation, we consider assigning two types of virtual springs between the agents: continuous and discontinuous springs. The way in which agents are interconnected by virtual springs is modeled by an undirected connected acyclic graph. The physical interconnection structure modeled by the graph (van der Schaft & Maschke, 2013) is assumed to be equal to the information exchange amongst agents (i.e., two agents exchange (local) information if there is a virtual spring assigned in between the agents). The way in which we define the springs fits well within the position-based formation control setting described in Arcak (2007) and Bai et al. (2011). Compared with existing literature (Vos et al., 2012; Vos, Scherpen, & van der Schaft, 2014) the major difference is the presence of discontinuity in the agents dynamics due to Coulomb friction. This discontinuity prevents continuous springs to achieve the formation control objectives which is the motivation behind proposing discontinuous springs for the control. For other results about discontinuous position-based formation control, we refer the reader to De Persis and Jayawardhana (2012) and Jafarian and De Persis (2015).

Main contribution. We present modeling and analysis of a network of planar heterogeneous dynamic point masses subject to Coulomb friction in the port-Hamiltonian framework. We show that continuous virtual springs fail to achieve a desired formation exactly. Moreover, we present a discontinuous distributed design to achieve the desired goals of the formation control problem. Both the network and the controller are modeled within the port-Hamiltonian framework which provides a clear physical interpretation of the results. It is worth noting that the use of nonsmooth analytical tools for formation keeping control in the port-Hamiltonian framework has not been studied before. Preliminary results of this work were presented in Jafarian, Vos, De Persis, van der Schaft, and Scherpen (2014) which considers the formation control of a group of homogeneous agents in $\mathbb{R}$ communicating over a star graph. In this paper, we generalize the results to the class of *tree graphs*. Furthermore, we provide a rigorous nonsmooth stability analysis of the closed-loop system.

The outline of this paper is as follows. First we recall some preliminaries on port-Hamiltonian systems, graph theory and analytical tools for nonsmooth systems. Section 3 presents a port-Hamiltonian model for the agents which are subject to Coulomb friction in $\mathbb{R}$ and $\mathbb{R}^2$. Section 4 continues with the control design and the closed-loop analysis for both continuous and discontinuous springs. Section 5 illustrates the effectiveness of the approach by simulation results. Finally, Section 6 concludes the paper.

Notation. For a square matrix $A \in \mathbb{R}^{n\times n}$, tr (A) denotes the trace of A, which is defined as tr $(A) = \sum_{i=1}^{n} a_{ii}$. The symbol $\times_{k=1}^{m} S_k$ denotes the Cartesian product $S_1 \times S_2 \times \cdots \times S_m$. The symbol $\bigcup_{r=1}^{n} S_r$ denotes the union $S_1 \cup S_2 \cup \cdots \cup S_n$. An empty set is denoted by $\emptyset$. Given a matrix M of real numbers, we denote by $\mathcal{R}(M)$ and $\mathcal{N}(M)$ the range and the null space, respectively. The symbols $\mathbf{1}, \mathbf{0}$ denote appropriately sized vectors or matrices of all 1 and 0 respectively. Sometimes the size of the matrix is explicitly given (i.e., $\mathbf{1}_n$ is the n-dimensional vector of all 1). I_p is the $p \times p$ identity matrix. Given two matrices A, B, the symbol $A \otimes B$ denotes the Kronecker product. block.diag $(A_1, \dots, A_n)$ denotes the block diagonal matrix such that A_i is its ith diagonal element. For a scalar function $H : \mathbb{R}^n \mapsto \mathbb{R}$, $\frac{\partial H}{\partial x}$ denotes the column vector of partial derivatives of the function $H(x)$ with respect to $x = (x_1, \dots, x_n)^T$. For vectors $a, b \in \mathbb{R}^n$, $a \cdot b$ denotes their inner product. Finally, the 2-norm and 1-norm of $a \in \mathbb{R}^n$ are denoted as $\|a\|$ and $\|a\|_1$ respectively.

2. Preliminaries

This section presents some preliminaries on the theory of port-Hamiltonian systems, graph theory and analytical tools for nonsmooth systems which are used in the remainder of this paper.

Port-Hamiltonian systems

The port-Hamiltonian framework is energy-based modeling framework which describes a large class of (nonlinear) multi-domain systems (Duindam et al., 2009; van der Schaft & Jeltsema, 2014). A port-Hamiltonian system consists of energy storing elements, energy dissipating elements and a Dirac structure which describes how the elements are interconnected in a power-preserving way. Furthermore, external ports are used to describe the interaction with the external systems like the control system and the environment.

There are several representations for port-Hamiltonian systems (van der Schaft & Jeltsema, 2014). In this work we are dealing with input-state-output port-Hamiltonian systems. Consider state $x \in \mathbb{R}^n$, skew-symmetric structure matrix $J(x) = -J^T(x) \in \mathbb{R}^{n\times n}$, positive semi-definite dissipation matrix $R(x) = R^T(x) \geq 0 \in \mathbb{R}^{n\times n}$, and let $H(x) : \mathbb{R}^n \to \mathbb{R}$ denote the Hamiltonian of the system. The Hamiltonian $H(x)$ is the sum of the kinetic and potential energy stored in the system. The systems considered in this work have two interaction ports (see Duindam et al., 2009): a control port (u, y) and a resistive port (u^r, y^r). Each port has two port-variables, being the inputs $u \in \mathbb{R}^{m_1}, u^r \in \mathbb{R}^{m_2}$ and outputs $y \in \mathbb{R}^{m_1}, y^r \in \mathbb{R}^{m_2}$. The product of the port-variables has the dimension of power and equals the energy flow through the port. The port-Hamiltonian dynamics (Duindam et al., 2009) are given by

$$\begin{aligned} \dot{x} &= J(x)\frac{\partial H}{\partial x}(x) + g(x)\,u + g^r(x)\,u^r \\ y &= g^T(x)\frac{\partial H}{\partial x}(x) \\ y^r &= g^{r\,T}(x)\frac{\partial H}{\partial x}(x), \end{aligned} \tag{1}$$

with input matrices $g(x) \in \mathbb{R}^{n\times m_1}, g^r(x) \in \mathbb{R}^{n\times m_2}$ corresponding to the control port and the resistive port respectively. The resistive port enables us to model the discontinuous Coulomb friction (see Section 3). A resistive element dissipates energy and hence the resistive port-variables satisfy $y^{r\,T} u^r \leq 0$.

Graph theory

The information exchange between agents is modeled by a connected and undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where node-set $\mathcal{V}$ corresponds to n agents and the edge-set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ corresponds to m virtual springs. In order to use the tools of graph theory, we assign a positive/negative label to each of the nodes connected by a link. The labeling of the nodes can be done in an arbitrary manner, and it does not affect the final results. Label one end of each edge in $\mathcal{E}$ with a positive sign and the other end with a negative sign. The incidence matrix B associated to $\mathcal{G}(\mathcal{V}, \mathcal{E})$ describes which nodes are coupled by an edge, and is defined as

$$b_{i\ell} = \begin{cases} +1 & \text{if node } i \text{ is at the positive side of edge } \ell \\ -1 & \text{if node } i \text{ is at the negative side of edge } \ell \\ 0 & \text{otherwise.} \end{cases}$$