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Minimax estimation with intermittent observations ☆

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ABSTRACT

This paper considers a problem of minimax (or H^∞) state estimation with intermittent observations. In this setting, the disturbance in the dynamical system and the sensor noise are controlled by adversaries, and the estimator receives the sensor measurements only sporadically, with the availability governed by an independent and identically distributed Bernoulli process. We cast this problem within the framework of stochastic zero-sum dynamic games. We first obtain a corresponding stochastic minimax state estimator (SMSE) and an associated generalized stochastic Riccati equation (GSRE) whose evolutions depend on two parameters: one that governs the random measurement arrivals and another one that quantifies the level of H^∞ disturbance attenuation. We then analyze the asymptotic behavior of the sequence generated by the GSRE in the expectation sense, and its weak convergence. Specifically, we obtain threshold-type conditions above which the sequence generated by the GSRE can be bounded both below and above in the expectation sense. Moreover, we show that under some conditions, the norm of the sequence generated by the GSRE converges weakly to a unique stationary distribution. Finally, we prove that when the disturbance attenuation parameter goes to infinity, our asymptotic results are equivalent to the corresponding results from the literature on Kalman filtering with intermittent observations. We provide simulations to illustrate the results.

1. Introduction

Networked control systems (NCSs) have taken center stage in recent years due to the widespread use of wireless communication and its applications to control systems—for example, command, control, communication, computing, and intelligence (C4I) in defense systems, unmanned aerial vehicles (UAVs), power and chemical plants, and automated systems. In contrast to classical feedback control systems, in NCSs communication networks are used between the controller and the plant, as well as between the sensors and the controller (Hespanha, Naghshtabrizi, & Xu, 2007).

Typical communication constraints that have been considered in the literature are delays, limitations on data rate, and presence of packet drops, where the latter is also referred to as data erasure (Hespanha et al., 2007). These three limitations are natural, because (1) delays are continually generated in communication while encoding, decoding, transmitting, and receiving signals, (2) every communication channel has a finite capacity under which the data can be handled reliably, and (3) the network is always subject to packet losses or link failures, especially in wireless communication.

In this paper, we study an optimal state estimation problem for linear time-invariant (LTI) systems when there are packet drops between the sensor and the estimator. The general configuration of the problem is captured in Fig. 1. In our model, the sensor measurements are only intermittently available to the estimator, where the intermittency of the observations is characterized by an independent and identically distributed (i.i.d.) Bernoulli process, $\{\beta_k\}$. If sensor measurements are lost, the estimator receives *nothing*. Therefore, the measurement arrivals are governed by the underlying probability distribution of β_k. Moreover, unlike the previous work on this topic, our model considers the case when the disturbance and the sensor noise, $\{w_k\}$ and $\{v_k\}$, are not necessarily stochastic processes, which are therefore treated as being controlled by adversaries in the estimation process. Under this setting, the optimal estimation problem is formulated under a worst-case scenario within the framework of stochastic zero-sum dynamic games.

To motivate this class of problems, consider a simple target estimation problem in a radar system, where estimation is performed

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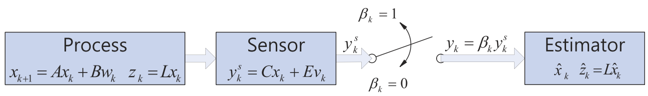

Fig. 1. System block diagram of minimax estimation with intermittent observations.

based on sensor signals that are reflected from the target (Li & Jilkov, 2005). Due to a large volume of clutter and unforeseeable weather conditions, reflected signals can be lost or delayed randomly. In this case, after a certain number of consecutive measurement losses, the number of false alarms and/or missing targets increases, which results in losing the target information. Now, to cope with this scenario, the main objective should be to achieve stability and performance of the estimator in terms of measurement losses. To be specific, we need to characterize the maximum tolerable measurement loss rate below which the estimator is able to track the target reliably. We should mention that this class of problems was studied first by Nahi (1969), and has been studied extensively in recent years as will be discussed later.

In addition to measurement packet losses, in many practical situations, it is not realistic that statistical information of the disturbance and the sensor noise is available, or even if they are available, there is a mismatch between statistical and true information on the disturbance and the sensor noise. To handle this situation, H^∞ estimation has been widely studied and applied in many practical applications, which deals with estimation under unknown arbitrary disturbance and the sensor noise controlled by adversaries (Başar & Bernhard, 1995; Nagpal & Khargonekar, 1991; Shaked & Theodor, 1992). There exist many applications of H^∞ estimation, see for example, Chen, Tsai, and Chen (2001), Labarre, Grivel, Najim, and Christov (2007), Lee, Chen, and Chen (2006) and Shen and Deng (1999).

The problem depicted in Fig. 1 has been studied extensively within the Kalman filtering framework, in which case certain statistical properties of the disturbance and the sensor noise, $\{w_k\}$ and $\{v_k\}$, need to be available to the estimator, as such, this can be seen as a special case of our model (this point will be made clear later in the paper).

Within the Kalman filtering framework, the first set of notable results were obtained by Sinopoli et al. (2004). There, the stochastic Kalman filter and the associated stochastic Riccati equation (or the stochastic error covariance matrix), say P_k, were obtained such that their processes are dependent on the entire measurement arrival information, $\{\beta_k\}$. Moreover, Sinopoli et al. (2004) showed that there is a critical value of the measurement loss rate beyond which the expected value of the error covariance matrix, $\mathbb{E}\{P_k\}$, is bounded. It was also shown that this critical value is a function of the unstable modes of the system, and can be analyzed in terms of lower and upper bounds.

The difference between the Kalman filter in Sinopoli et al. (2004) and the Markov jump linear estimator (MJLE) in Costa and Guerra (2002) is that the latter is dependent only on the current measurement arrival information β_k. It was shown in Sinopoli et al. (2004) that the Kalman filter in their paper is optimal over all possible estimators, and thus provides better estimation performance than the MJLE.

The results obtained in Sinopoli et al. (2004) were extended in many different directions, with some related references being Censi (2011), Epstein, Shi, Tiwari, and Murray (2008), Huang and Dey (2007), Kar, Sinopoli, and Moura (2012), Kluge, Reif, and Brokate (2010), Liu and Goldsmith (2004), Mo and Sinopoli (2008, 2011, 2012), Plarre and Bullo (2009) and Shi, Epstein, and Murray (2010). Moreover, instead of $\mathbb{E}\{P_k\}$, other performance metrics, or a correlated intermittent observation case were studied in Censi (2011), Epstein et al. (2008), Huang and Dey (2007), Kar et al. (2012), Mo and Sinopoli (2012) and Shi et al. (2010), to provide other perspectives on the error covariance matrix, P_k.

Although there are significant progresses on the intermittent observation problem within the Kalman filtering framework, it has not yet been addressed thoroughly through the worst-case or H^∞ approach. Some relevant results were obtained in De Souza and Fragoso (1997), Gao and Chen (2007), Goncalves, Fioravanti, and Geromel (2009), Sahebsara, Chen and Shah (2008) and Wang, Yang, Ho, and Liu (2006), where the different sets of linear matrix inequalities were derived for the H^∞ performance. These results, however, are related more to the theory of MJLEs and are therefore suboptimal, since the estimators are restricted to be time-invariant and obtained under the current measurement arrival information β_k.

Taking the classes of intermittent estimation problems, as described above, a step further, we study in this paper the problem of minimax[2] state estimation with intermittent observations for LTI systems.

By formulating the problem within the framework of stochastic zero-sum dynamic games, we first obtain a stochastic minimax state estimator (SMSE) and an associated generalized stochastic Riccati equation (GSRE) that are time-varying and random, and are dependent on the sequence of the random measurement arrival information $\{\beta_k\}$ and the H^∞ disturbance attenuation parameter γ. We then identify an existence condition for the SMSE in terms of the GSRE and γ. We also show that under that existence condition, the SMSE is able to attenuate arbitrary disturbances within the level of γ. Moreover, we show that for the least disturbance attenuation case (that is, as $\gamma \to \infty$), the SMSE and the GSRE converge, respectively, to the Kalman filter and its stochastic Riccati equation $\{P_k\}$ in Sinopoli et al. (2004).

The second objective of this paper is to analyze the asymptotic behavior of the SMSE. In particular, we prove boundedness of the sequence generated by the GSRE in the expectation sense, and also show its weak convergence. More specifically, we first show that under the existence condition, there exist a critical value of the measurement loss rate and a critical value for the disturbance attenuation parameter beyond which the expected value of the sequence generated by the GSRE can be bounded both below and above. Second, we prove that under the existence condition, the norm of the sequence generated by the GSRE converges weakly to a unique stationary distribution. For both cases, we show that when $\gamma \to \infty$, the corresponding asymptotic results are equivalent to that in Kar et al. (2012) and Sinopoli et al. (2004). We also demonstrate by simulations that the SMSE outperforms the stationary and suboptimal H^∞ MJLE in Goncalves et al. (2009), as did for the Kalman filtering problem in Sinopoli et al. (2004).

The structure of the paper is as follows. In Section 2, we formulate the problem of minimax estimation with intermittent observations. In Section 3, we obtain the SMSE and GSRE, and characterize the existence condition. In Section 4, we analyze the asymptotic behavior of the GSRE. In Section 5, we present simulation results. We end the paper with the concluding remarks of Section 6. Supporting lemmas and some useful results on Kalman filtering with intermittent observations are provided in Appendices.

Notation

$\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the spaces of n-dimensional and $m\times n$-dimensional real-valued vectors and matrices. $\mathbb{S}^n_{\succeq 0}$ (resp.

[2] In this paper, we will be using the qualifiers "H^∞" and "minimax" estimation interchangeably (Başar & Bernhard, 1995).