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Brief paper

An improved stability condition for Kalman filtering with bounded Markovian packet losses[☆]

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ABSTRACT

In this paper, we consider the peak-covariance stability of Kalman filtering subject to packet losses. The length of consecutive packet losses is governed by a time-homogeneous finite-state Markov chain. We establish a sufficient condition for peak-covariance stability and show that this stability check can be recast as a linear matrix inequality (LMI) feasibility problem. Compared with the literature, the stability condition given in this paper is invariant with respect to similarity state transformations; moreover, our condition is proved to be less conservative than the existing results. Numerical examples are provided to demonstrate the effectiveness of our result.

1. Introduction

Networked control systems are closed-loop systems, wherein sensors, controllers and actuators are interconnected through a communication network. In the last decade, advances of modern control, micro-electronics, wireless communication and networking technologies have given birth to a considerable number of networked control applications.

In networked control systems, state estimation such as using a Kalman filter is necessary whenever precise measurement of the system state cannot be obtained. When a Kalman filter is running subject to intermittent observations, the stability of the estimation error is affected by not only the system dynamics but also by the statistics of the packet loss process. The stability of Kalman filtering with packet drops has been intensively studied in the literature. In Sinopoli et al. (2004), Plarre and Bullo (2009), Mo and Sinopol (2010), Shi, Epstein, and Murray (2010) and Kar, Sinopoli, and Moura (2012), independently and identically distributed (i.i.d.) Bernoulli packet losses have been considered. Some other research works assume the packet drops, due to the Gilbert–Elliott channel (Gilbert, 1960; Elliott, 1963), are governed by a time-homogeneous Markov chain. Huang and Dey (2007) introduced the notion of peak covariance, which describes an upper envelope of the sequence of error covariance matrices for the case of an unstable scalar system. They focused on its stability with Markovian packet losses and gave a sufficient stability condition. The stability condition was further improved in Xie and Xie (2007) and Xie and Xie (2008). In Wu, Shi, Anderson, and Johansson (0000), the authors proved that the peak-covariance stability implies mean-square stability for general random packet drop processes, if the system matrix has no defective eigenvalues on the unit circle. In addition to the peak-covariance stability, the mean-square stability was considered for some classes of linear systems in Mo and Sinopoli (2012), You, Fu, and Xie (2011), and weak convergence of the estimation error covariance was studied in Xie (2012).

In the aforementioned packet loss models, the length of consecutive packet losses can be infinitely large. In contrast, some works also considered bounded packet loss model, whereby the length of consecutive packet losses is restricted to be less than a finite integer. A real example of bounded packet losses is the WirelessHART (Wireless Highway Addressable Remote Transducer) protocol, the state-of-the-art wireless communication solution for process automation applications. In WirelessHART, there are two types of time slots: one is the dedicated time slot allocated to a specific field device for time-division multiple-access (TDMA) based transmission and the other is the shared time slot for contention-based

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communication. A contiguous group of time slots during a constant period of time forms a superframe, within which every node is guaranteed at least one time slot for data communication. Various networked control problems with bounded packet loss models have been studied, e.g., Wu and Chen (2007) and Xiong and Lam (2007); while the stability of Kalman filtering for this kind of models was rarely discussed. In Xiao, Xie, and Fu (2009), the authors gave a first attempt to the stability issue related to the Kalman filtering with bounded Markovian losses. They provided a sufficient condition for peak-covariance stability, the stability notion studied in Huang and Dey (2007), Xie and Xie (2007) and Xie and Xie (2008). Their result has established a connection between peak-covariance stability, the dynamics of the underlying system and the probability transition matrix of the underlying packet-loss process. In this paper, we consider the same problem as in Xiao et al. (2009) and improve the stability condition thereof. The main contributions of this work are summarized as follows:

(1) We present a sufficient condition for peak-covariance stability of the Kalman filtering subjected to bounded Markovian packet losses (Theorem 1). Different from that of Xiao et al. (2009), this stability check can be recast as a linear matrix inequality (LMI) feasibility problem (Proposition 1).
(2) We compare the proposed condition with that of Xiao et al. (2009). We show both theoretically and numerically that the proposed stability condition is invariant with respect to similarity state transformations, while the one given in Xiao et al. (2009) may generate opposite conclusions under different similarity transformations. Moreover, the analysis also suggests that our condition is less conservative than the former one.

The remaining part of the paper is organized as follows. Section 2 presents the mathematical models of the system and packet losses, and introduces the preliminaries of Kalman filtering. Section 3 provides the main results. Comparison with Xiao et al. (2009) and numerical examples are presented in Section 4. Some concluding remarks are drawn in the end.

Notations. $\mathbb{N}$ is the set of positive integers and $\mathbb{C}$ is the set of complex numbers. $\mathbb{S}_+^n$ is the set of n by n positive semi-definite matrices over the field $\mathbb{C}$. For a matrix $X \in \mathbb{C}^{n\times n}$, $\sigma(X)$ denotes the spectrum of X, i.e., $\sigma(X) = \{\lambda : \det(\lambda I - X) = 0\}$, and $\rho(X)$ denotes the spectrum radius of X, X^*, X' and $\overline{X}$ are the Hermitian conjugate, transpose and complex conjugate of X, respectively. $\|\cdot\|$ means the L_2-norm on $\mathbb{C}^n$ or the matrix norm induced by L_2-norm. The symbol $\otimes$ represents the Kronecker product operator of two matrices. For any matrices A, B, C with compatible dimensions, we have $\mathrm{vec}(ABC) = (C' \otimes A)\mathrm{vec}(B)$, where $\mathrm{vec}(\cdot)$ is the vectorization of a matrix. Moreover, the indicator function of a subset $\mathcal{A} \subset \Omega$ is a function $\mathbf{1}_{\mathcal{A}} : \Omega \to \{0, 1\}$ where $\mathbf{1}_{\mathcal{A}}(\omega) = 1$ if $\omega \in \mathcal{A}$, otherwise $\mathbf{1}_{\mathcal{A}}(\omega) = 0$. The symbol $\mathbb{E}[\cdot]$ (resp., $\mathbb{E}[\cdot|\cdot]$) represents the expectation (resp., conditional expectation) of a random variable.

2. Problem setup

2.1. System model

Consider the following discrete-time LTI system:

$$x_{k+1} = Ax_k + w_k, \tag{1a}$$

$$y_k = Cx_k + v_k, \tag{1b}$$

where $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$, $x_k \in \mathbb{R}^n$ is the process state vector, $y_k \in \mathbb{R}^m$ is the observation vector, $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are zero-mean Gaussian random vectors with $\mathbb{E}[w_k w_j{}'] = \delta_{kj}Q$ $(Q \geq 0)$, $\mathbb{E}[v_k v_j{}'] = \delta_{kj}R$ $(R > 0)$, $\mathbb{E}[w_k v_j{}'] = 0$ $\forall j, k$. Note that δ_{kj} is the Kronecker delta function with $\delta_{kj} = 1$ if $k = j$ and $\delta_{kj} = 0$ otherwise. The initial state x_0 is a zero-mean Gaussian random vector that is uncorrelated to w_k and v_k, with covariance $\Sigma_0 \geq 0$. It can be seen that, by applying a similarity transformation, the unstable and stable modes of the LTI system can be decoupled. An open-loop prediction of the stable mode always has a bounded estimation error covariance, therefore, this mode does not play any key role in the problem considered below. Without loss of generality, all eigenvalues of A are assumed to have magnitudes not less than 1. We also assume that (A, C) is observable and $(A, Q^{1/2})$ is controllable. We introduce the definition of the observability index of (A, C), which is taken from Antsaklis and Michel (2006).

Definition 1. The observability index $\mathrm{I_o}$ is defined as the smallest integer such that $[C', A'C', \ldots, (A^{\mathrm{I_o}-1})'C']'$ has rank n. If $\mathrm{I_o} = 1$, the system (A, C) is called one-step observable.

2.2. Bounded Markovian packet-loss process

In this paper, we consider the estimation scheme, where the raw measurements $\{y_k\}_{k\in\mathbb{N}}$ of the sensor are transmitted to the estimator over an erasure communication channel: packets may be randomly dropped or successively received by the estimator. Denote by a random variable $\gamma_k \in \{0, 1\}$ whether or not y_k is received at time k. If $\gamma_k = 1$, it indicates that y_k arrives error-free at the estimator; otherwise $\gamma_k = 0$. Whether γ_k equals 0 or 1 is assumed to have been known by the estimator before time $k + 1$. In order to introduce the packet loss model, we further define a sequence of stopping times, the time instants at which packets are received by the estimator:

$$t_1 \triangleq \min\{k : k \in \mathbb{N}, \gamma_k = 1\},$$
$$t_2 \triangleq \min\{k : k > t_1, \gamma_k = 1\},$$
$$\vdots \tag{2}$$
$$t_j \triangleq \min\{k : k > t_{j-1}, \gamma_k = 1\}, \tag{3}$$

where we assume $t_0 = 0$ by convention. The packet-loss process, τ_j, is defined as

$$\tau_j \triangleq t_j - t_{j-1} - 1.$$

As for the model of packet losses, we assume that the packet-loss process $\{\tau_j\}_{j\in\mathbb{N}}$ is modeled by a time-homogeneous ergodic Markov chain, where $\mathcal{S} = \{0, \ldots, s\}$ is the finite-state space of the Markov chain with s being the maximum length of consecutive lost packets allowed. Here the Markov chain is characterized by a known transition probability matrix $\mathbf{\Pi} \triangleq [\pi_{ij}]_{i,j\in\mathcal{S}}$ in which

$$\pi_{ij} \triangleq \mathbb{P}(\tau_{k+1} = j | \tau_k = i) \geq 0. \tag{4}$$

Denote the initial distribution as $p \triangleq [p_0, \ldots, p_s]$, where $p_j = \mathbb{P}(\tau_1 = j)$.

2.3. Kalman filtering with packet losses

Sinopoli et al. (2004) shows that, when performed with intermittent observations, the optimal linear estimator is a modified Kalman filter. The modified Kalman filter is slightly different from the standard one in that only time update is performed in the presence of the lost packet. Define the minimum mean-squared error estimate and the one-step prediction at the estimator respectively as $\hat{x}_{k|k} \triangleq \mathbb{E}[x_k|\gamma_1 y_1, \ldots, \gamma_k y_k]$ and $\hat{x}_{k+1|k} \triangleq \mathbb{E}[x_{k+1}|\gamma_1 y_1, \ldots, \gamma_k y_k]$. Let $P_{k|k}$ and $P_{k+1|k}$ be the corresponding estimation and prediction error covariance matrices, i.e.,

$$P_{k|k} \triangleq \mathbb{E}[(x_k - \hat{x}_{k|k})(\cdot)'|\gamma_1 y_1, \ldots, \gamma_k y_k]$$
$$P_{k+1|k} \triangleq \mathbb{E}[(x_{k+1} - \hat{x}_{k+1|k})(\cdot)'|\gamma_1 y_1, \ldots, \gamma_k y_k].$$