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Brief paper

Distributed output regulation for a class of nonlinear multi-agent systems with unknown-input leaders ☆

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ABSTRACT

In this paper, a distributed output regulation problem is formulated for a class of uncertain nonlinear multi-agent systems subject to local disturbances. The formulation is given to study a leader-following problem when the leader contains unknown inputs and its dynamics is different from those of the followers. Based on the conventional output regulation assumptions and graph theory, distributed feedback controllers are constructed to make the agents globally or semi-globally follow the uncertain leader even when the bound of the leader's inputs is unknown to the followers.

1. Introduction

The past decade has witnessed a rapid development in the field of multi-agent systems and fruitful results have been achieved for the leader-following coordination problem. In recent years, distributed output regulation of multi-agent systems has been proposed to provide a general framework for leader-following consensus in linear or nonlinear cases (Dong & Huang, 2014; Hong, Wang, & Jiang, 2013). Internal model approach, developed to solve the conventional output regulation (Huang, 2004; Isidori, Marconi, & Serrani, 2003; Yang & Huang, 2010), was effectively used for the distributed design, especially for nonlinear agents. For example, a cooperative output regulation problem was considered for a class of nonlinear uncertain multi-agent systems with unity relative degree in Su and Huang (2013), while agents in the output-feedback form were considered with an autonomous leader and no-loop graph (Ding, 2013).

It may be restrictive or unpractical if we always consider an autonomous leader without unknown inputs, especially in the case when the leader is an uncooperative target or contains unmodeled uncertainties. Therefore, it is necessary to study multi-agent control when the leader contains (unknown) inputs. In fact, the generalized output regulation (GOR) problem to track an exosystem (or a leader) with external inputs was discussed in many publications including Ramos, Celikovsky, and Kucera (2004) and Saberi, Stoorvogel, and Sannuti (2001). On the other hand, a distributed problem was investigated when the agent dynamics are double integrators to track a leader with an unknown but bounded acceleration in Cao and Ren (2012), and then a similar design was given in Li, Liu, Ren, and Xie (2013) by assuming that the leader and followers share the same dynamics. To our best of knowledge, there are no general results on nonlinear multi-agent control when the unknown-input leader and the followers have different dynamics with external disturbances.

The objective of our paper is to study distributed output regulation for leader-following multi-agent systems with an unknown-input leader, whose dynamics is nonlinear and may differ from those of the followers. The contribution of the work is at least twofold:

- We extend the distributed output regulation to the case when the leader contains unknown inputs and has dynamics different from those of the non-identical followers with (unbounded) local disturbances, and provide distributed controls to solve

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this problem in different cases. The results are consistent with the existing output regulation results when the leader does not have unknown inputs and the disturbances are bounded (e.g. Dong & Huang, 2014; Su & Huang, 2013).
- We extend the conventional GOR to its distributed version for multi-agent systems with an unknown-input leader. Moreover, both global and semi-global results are obtained for nonlinear agents with unity relative degree, while only local results were obtained for a conventional (single-agent) case in Ramos et al. (2004).

The rest of this paper is organized as follows. Preliminaries and problem formulation are given in Section 2. Then our main results are presented in Section 3, followed by simulations in Section 4. Finally, concluding remarks are presented in Section 5.

Notations: Let $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^n_M = \{s \in \mathbb{R}^n \mid -M \le s_i \le M, i = 1, \ldots, n\}$ for a constant $M > 0$. For a vector x, $\|x\|$ (or $\|x\|_\infty$) denotes its Euclidean norm (or infinite norm). $\mathrm{diag}\{b_1, \ldots, b_n\}$ denotes an $n \times n$ diagonal matrix with diagonal elements b_i $(i = 1, \ldots, n)$; $\mathrm{col}(a_1, \ldots, a_n) = [a_1^T, \ldots, a_n^T]^T$ for column vectors a_i $(i = 1, \ldots, n)$. A continuous function $\alpha: [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$; It belongs to class $\mathcal{K}_\infty$ if it belongs to class $\mathcal{K}$ with $a = \infty$ and $\lim_{s\to\infty} \alpha(s) \to \infty$.

2. Problem formulation

Consider a group of $n + 1$ agents with one leader (regarded as node 0) as follows:

$$\dot{v} = p(v) + q(v)w(t), \qquad y_0 = r(v, \mu) \tag{1}$$

where $v \in \mathbb{R}^{n_v}$ is the leader's state, and $w(t) \in \mathbb{R}^{n_w}$, $y_0 \in \mathbb{R}$ are its input and output, respectively. Here $\mu \in \mathbb{R}^{n_\mu}$ is an uncertain parameter vector, and $w(t)$ is continuous satisfying $\|w(t)\|_\infty \le l$ with a constant $l > 0$. The other n (non-identical) agents are followers described by

$$\begin{cases} \dot{z}_i = f_i(z_i, y_i, \mu) \\ \dot{y}_i = g_i(z_i, y_i, \mu) + b_i u_i + d_i, \quad i = 1, \ldots, n \end{cases} \tag{2}$$

where $z_i \in \mathbb{R}^{n_{z_i}}$, $y_i \in \mathbb{R}$, $d_i \in \mathbb{R}$, $b_i > 0$. Without loss of generality, we take $b_i = 1$ and assume all functions f_i, g_i, p, q, r are smooth with $f_i(0, 0, \mu) = 0$, $g_i(0, 0, \mu) = 0$, $p(0) = 0$, $r(0, \mu) = 0$. d_i is the local disturbance of agent i governed by

$$\dot{\omega}_i = S_i \omega_i, \qquad d_i = D_i(\mu)\omega_i. \tag{3}$$

As usual, we assume $S_i \in \mathbb{R}^{n_{\omega_i} \times n_{\omega_i}}$ has no eigenvalues with negative real parts (Huang, 2004).

Clearly, the first-order nonlinear agent in Liu, Xie, Ren, and Wang (2013) is a special case of (2), and system (2) was also considered to solve an output consensus problem for the exosystem without any inputs in Ding (2013).

The interaction topology among these agents can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0, 1, \ldots, n\}$ is the set of nodes and $\mathcal{E}$ is the set of arcs. (i, j) denotes an arc leaving from node i and entering node j (Godsil & Royle, 2001). A walk in graph $\mathcal{G}$ is an alternating sequence $i_1 e_1 i_2 e_2 \cdots e_{k-1} i_k$ of nodes i_l and arcs $e_m = (i_m, i_{m+1}) \in \mathcal{E}$ for $l = 1, 2, \ldots, k$. If there exists a walk from node i to node j, then node i is said to be reachable from j. Define the neighbor set of agent i as $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$ for $i = 1, \ldots, n$. A weighted adjacency matrix of $\mathcal{G}$ is denoted by $A = [a_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$, where $a_{ii} = 0$ and $a_{ij} \ge 0$ ($a_{ij} > 0$ if $(j, i) \in \mathcal{E}$). A graph is said to be undirected if $a_{ij} = a_{ji}$ $(i, j = 0, 1, \ldots, n)$. The Laplacian $L = [l_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$ of graph $\mathcal{G}$ is defined as $l_{ii} = \sum_{j\ne i} a_{ij}$ and $l_{ij} = -a_{ij}$ $(j \ne i)$. Denote $\bar{\mathcal{G}}$ as the induced subgraph of $\mathcal{G}$ associated with the n followers. The following assumption has been widely used in coordination of multi-agent systems (Hong, Hu, & Gao, 2006; Su & Huang, 2013).

Assumption 1. The leader (node 0) is reachable from any other node of $\mathcal{G}$ and the induced subgraph $\bar{\mathcal{G}}$ is undirected.

Given a communication graph $\mathcal{G}$, denote $H \in \mathbb{R}^{n\times n}$ as the sub-matrix of the Laplacian L by deleting its first row and first column. By Lemma 3 in Hong et al. (2006), H is positive definite under Assumption 1. The distributed control law can be constructed as follows:

$$\begin{aligned} u_i &= k_i(\xi_i, y_i - y_j, j \in \mathcal{N}_i), \\ \dot{\xi}_i &= h_i(\xi_i, y_i - y_j, j \in \mathcal{N}_i) \end{aligned} \tag{4}$$

where $\xi_i \in \mathbb{R}^{n_{\xi_i}}$ with a nonnegative integer n_{ξ_i} and functions $k_i(\cdot)$, $h_i(\cdot)$ to be designed later.

To handle this nonlinear multi-agent system with an unknown-input leader, we formulate the problem as the *distributed generalized output regulation problem* or simply *distributed regulation problem*. It is said to be (globally) solved for systems (1)–(3) with a given graph $\mathcal{G}$, if we can find a distributed control law (4), such that, for any $(z_i(0), y_i(0)) \in \mathbb{R}^{n_{z_i}+1}$, $\mu \in \mathbb{R}^{n_\mu}$, $\xi_i(0) \in \mathbb{R}^{n_{\xi_i}}$, $v(0) \in \mathbb{R}^{n_v}$, $\omega_i(0) \in \mathbb{R}^{n_{\omega_i}}$, the trajectory of the closed-loop system, composed of (1), (2), and (4), is well-defined for all $t > 0$, and moreover,

$$\lim_{t\to\infty} e_i(t) = 0, \quad e_i = y_i - y_0, \ i = 1, \ldots, n. \tag{5}$$

Our problem is said to be semi-globally solved if, for any given $M > 0$, we can find a control law (4) with a constant $\bar{M} \ge 0$, such that, for any initial condition $(z_i(0), y_i(0)) \in \mathbb{R}^{n_{z_i}+1}_M$, $\mu \in \mathbb{R}^{n_\mu}_M$, $\xi_i(0) \in \mathbb{R}^{n_{\xi_i}}_{\bar{M}}$, $v(0) \in \mathbb{R}^{n_v}_M$ and $\omega_i(0) \in \mathbb{R}^{n_{\omega_i}}_M$, the trajectory of the closed-loop system is well-defined for all $t > 0$ and (5) holds.

Remark 1. When $n = 1$, our problem becomes GOR studied in Ramos et al. (2004) and Saberi et al. (2001). Here we seek non-local output feedback control for nonlinear systems of the form (2), while only local results were obtained in Ramos et al. (2004) requiring the exosystem's state.

Remark 2. Because of the unknown expression or type of $w(t)$, adaptive IM discussed in Su and Huang (2013) fails to solve our problem even with a time-varying one like in Yang and Huang (2010). In fact, our problem can be viewed as a distributed version of GOR to handle those exosystems (or leaders) with unknown inputs, which certainly extends the existing multi-agent output regulation formulation when the leaders have no unknown inputs (Hong et al., 2013; Su & Huang, 2013).

The following assumption was used for GOR of nonlinear systems (see Ramos et al., 2004).

Assumption 2. There exist two class $\mathcal{K}$ functions $\alpha_0(\cdot)$ and $\gamma_0(\cdot)$ such that

$$\|v(t)\| \le \alpha_0(\|v(0)\|) + \gamma_0(\|w(t)\|_\infty).$$

Clearly, $v(t)$ is bounded by $\bar{b}_1 = \alpha_0(\|v(0)\|) + \gamma_0(l)$ and $\bar{b}_2 = \max_{v\in\mathbb{R}^{n_v}_{\bar{b}_1}}\{\frac{\partial r}{\partial v}q(v)\}$ is well-defined under this assumption. Moreover, system (1) is Lyapunov stable at $v = 0$ when $w = 0$ with the neutrally stable exosystem for nonlinear output regulation as one of its special cases. In addition, if the origin of the unforced leader (i.e. $w(t) = 0$) is globally exponentially stable, Assumption 2 is satisfied for any $w(t)$ when $\|q(v)\|$ is bounded.

Similar to the output regulation problem, the solvability of regulator equations plays a key role in the study of nonlinear GOR. Therefore, we give the following assumption for the solution of regulator equations.